7.7 Numerical Integration

Trapezoid Rule

To approximate $\int_a^b f(x)\,dx$ with trapezoids and n subdivisions at $\{x_0, x_1, x_2, \cdots, x_n\}$, let $\Delta x = \frac{b-a}{n}$. Then...

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

The error is : $E_n \le \frac{K(b-a)^3}{12n^2}$ where $\left| f''(c) \right| \le K$ for all $c \in [a, b]$

Example 1 a Approximate $\int_2^6 x^2\,dx$ using trapezoids and $n = 5$ subdivisions.

Example 1 b Estimate the maximum error, and calculate the actual error, as well as the relative error.

Simpson's Rule or The Parabolic Rule (n is even)

To approximate $\int_a^b f(x)\,dx$ using Simpson's rule and n subdivisions $\{x_0, x_1, x_2, \cdots, x_n\}$, let $\Delta x = \frac{b-a}{n}$. Then...

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

The error is : $E_n \le \frac{K(b-a)^5}{180n^4}$ where $\left| f^{(4)}(c) \right| \le K$ for all $c \in [a, b]$

Example 2 a Approximate $\int_1^3 \frac{1}{1+x}\,dx$ using 6 subdivisions.

Example 2 b Estimate the maximum possible error.

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Example 3 Approximate $\int_0^2 3\cos(x)\, e^{-x^3}\, dx$ using Simpson's Rule and $n = 6$.

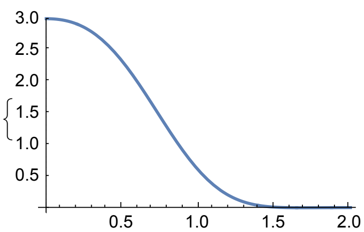
Mathematica, after a little while, gives up on an exact analytical value:

```
In[7]:= Integrate[3 Cos[x] e^(-x^3), {x, -0, 2}]
```

Out[7]= $\int_0^2 3\, e^{-x^3}\, \mathrm{Cos}[x]\, dx$

Using **NIntegrate,** we do get an approximation. But what does it actually do when evaluating the integral?!

```
In[9]:= {Plot[3 Cos[x] e^(-x^3), {x, 0, 2}, PlotRange → {-.1, 3}], NIntegrate[3 Cos[x] e^(-x^3), {x, 0, 2}]}
```

Out[9]= {

, 2.21499}

Example 4 Find the number of subdivisions, n, needed to evaluate $\int_2^8 (x^3 - 12x^2 - 5)\, dx$ accurate to 0.001 using:

a) Trapezoid rule
b) Simpson's Rule

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