

7.5 Strategy for Integration / 7.6 Tables

Some Basic Guidelines and Suggestions

1. Simplify the integrand if possible.
2. Look for an obvious substitution.
3. Classify the integrand according to its form (*trigonometric function, rational function, integration by parts, radical function, etc.*)
4. Try again, and again...

Example 1 What technique should be used on each of the following expressions? Integrate those that are possible.

a) $\int \frac{1}{x^2+1} dx$ b) $\int \frac{x}{x^2+1} dx$ c) $\int \frac{x^2}{x^2+1} dx$ d) $\int \frac{1}{x^2-1} dx$ e) $\int \frac{1}{(x+1)^2} dx$ f) $\int \frac{1}{\sqrt{1-x^2}} dx$ g) $\int \frac{1}{1+\sqrt{x}} dx$ h) $\int \frac{1}{\sqrt{1-x^3}} dx$

Find the following anti-derivatives:

Example 2 $\int \frac{x}{\sqrt{3-x^4}} dx$

Example 3 $\int \frac{x}{x^4+x^2+1} dx$

Example 4 $\int x \sin^{-1}(x) dx$

Example 5 $\int \frac{1}{\sqrt{4x^2 - 4x - 3}} dy$

Example 6 $\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$

Example 7 $\int \frac{2x^4 + 3}{x^3 - x^2 + 4x - 4} dx$

Example 8 $\int \ln(x^2 - 1) dx$

Example 9 $\int \frac{1}{1 - \cos(4x)} dx$

Example 10 $\int \frac{\sin(6x)}{\sqrt{3 - 4\sin(3x)}} dx$

Hint: from Tables #55 $\int \frac{u}{\sqrt{a+bu}} du = \frac{2}{3b^2} (bu - 2a) \sqrt{a+bu} + C$

Answers:

2: $\frac{1}{2} \sin^{-1}\left(\frac{x^2}{\sqrt{3}}\right) + C$ 3: $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2}{\sqrt{3}}\left(x^2 + \frac{1}{2}\right)\right) + C$ 4: $\frac{1}{4}[(2x^2 - 1)\sin^{-1}(x) + x\sqrt{1-x^2}] + C$

5: $\frac{1}{2} \ln \left| 2x - 1 + \sqrt{4x^2 - 4x - 3} \right| + C$ 6: $\frac{2}{3}[(x+1)^{3/2} - x^{3/2}] + C$

7: $2x + x^2 - \frac{7}{2} \tan^{-1}\left(\frac{x}{2}\right) + \ln(1-x) - \frac{7}{2} \ln(4+x^2) + C$ 8: $-2x - \ln(1-x) + \ln(x+1) + x \ln(x^2 - 1) + C$

9: $\frac{-1}{4} \cot(2x) + C$