

7.4 Partial Fractions

Rules for Decomposing Rational Expressions

1. Distinct linear factors: $\frac{P(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$
2. Distinct irreducible quadratic factors: $\frac{P(x)}{(ax^2+bx+c)(dx^2+ex+f)} = \frac{Ax+B}{ax^2+bx+c} + \frac{Cx+D}{dx^2+ex+f}$
3. Distinct linear and irreducible quadratic factors: $\frac{P(x)}{(ax+b)(cx^2+dx+e)} = \frac{A}{ax+b} + \frac{Bx+C}{cx^2+dx+e}$
4. Repeated factors: $\frac{P(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \dots + \frac{N}{(ax+b)^n}$

Example 1 Evaluate: $\int \frac{2x+14}{x^2+2x-8} dx$

Example 2 Evaluate: $\int \frac{x^2+10x+1}{x^3+x^2-x-1} dx$

Example 3Evaluate: $\int \frac{7x^2 - 10x + 10}{2x^3 - 4x^2 + 4x - 8} dx$ **Example 4**

Write down the possible partial decomposition:

$$\frac{52488}{3x(x+2)^3(x^2+2x-9)^2}$$

Mathematica and Partial Fractions

Apart $\left[\frac{52488}{3x(x+2)^3(x^2+2x-9)^2} \right]$ // TraditionalForm

$$\frac{16(x-3)}{x^2+2x-9} - \frac{24(2x-9)}{(x^2+2x-9)^2} + \frac{27}{x} - \frac{43}{x+2} - \frac{6}{(x+2)^2} - \frac{108}{(x+2)^3}$$