6.4 Application: Work

Two Key Formulas

Newtons' Second Law of Motion states that force is mass times acceleration, $F = m\,a$, or $F = m\,\frac{d^2 s}{ds^2}$, where s is the displacement function.

Also, **work** is defined as *force* × *distance*, assuming the force is in the same direction as the displacement: $w = F\,d$.

Example 1 a) Find the work done in raising a 5 pound book a distance of 3 feet.

b) Find the work done in raising a 3 kg book a distance of 40 cm.

If the force is not constant over the entire distance, we can approximate a very small distance by Δx, and assume the force $F(x)$ is constant on this interval. Summing all the approximations and taking the limit as the distance $\Delta x \to 0$ gives

$$W = \int F(x)\,dx$$

Example 2 A 4-N force is required to stretch a spring 16 cm beyond its natural length. Calculate the work done in stretching the spring an additional 10 cm. Note: Hooke's Law states the force, F, acting on a spring is directly proportional to the displacement x, or $F = k\,x$.

Example 3 A water tank is in the shape of an inverted right circular cone with height 12 feet and diameter 6 feet. The depth of the water is 9-feet. Calculate the work done in pumping all the water over the top of the tank. Note: $\rho_{H_2O} = 62.4\ \text{lb}/\text{ft}^3$.

Example 4 A 50-ft rope weighing 4-oz per foot is hanging over the top of a building. Calculate the work done in pulling the entire rope to the top?

Example 5 A 10-kg monkey is hanging from a limb of a tree attached to a 20-meter chain with linear density 0.2 kg/meter. Calculate the work done by the monkey in climbing to the limb?

Example 6 A leaky bucket weighing 25 pounds can hold 8-cubic feet of water, but leaks at a rate of $0.1\ \text{ft}^3/\text{second}$. The bucket is used to get water from a 40-ft well, and is attached to a rope weighing 0.4 lbs/ft. If the bucket can be raised at a rate of 2 ft/sec, calculate the work done in raising an initially full bucket of water to the top of the well?

Example 7 A space capsule with a mass of 2500 kg is propelled to an altitude of 350 km above the surface of the earth. How much work is done against gravity? Assume the earth is a sphere of radius 6000 km and that the force of gravity is inversely proportional to the square of the distance from the center of the earth. Thus, the lifting force is 2500(9.8) N when the distance is 6000 km.

Note: A pound is a measure of force; a slug is a measure of mass.
A Newton is a measure of force; a kg is a measure of mass.
Acceleration due to gravity: $9.8\ m/s^2$, or $32\ \text{ft}/s^2$.
Force = mass × acceleration
A Joule is a measure of work and $1\,J = 1\,N\cdot m$