6.2 Part II: Volumes of Revolutions

The Disk Method

A continuous function f (≥ 0) revolved about the x-axis on the interval $[a, b]$ creates a solid with volume

$$V = \int_a^b \pi f(x)^2 \, dx$$

Example 1 Find the volume of the solid created by revolving $f(x) = x^2 + 2$ on the Interval $[-1, 2]$ about the x-axis.

Example 2 Find the volume of the solid in the first quadrant bounded by $y = x^3 + 1$ and $y = 9$, about the y-axis.

The Washer Method

Let f and g be two continuous functions and $f \geq g \geq 0$ on the interval $[a, b]$. The volume of the solid created by revolving the bounded region between f and g about the x-axis on the interval $[a, b]$ is

$$V = \int_a^b \pi \left[f(x)^2 - g(x)^2 \right] dx$$

Example 3 Find the volume of rotation obtained by revolving the region bounded by $y = \frac{1}{x}$ and $y = x^2$ on the interval $[1, 4]$ about the x-axis.

Example 4 The region in the first quadrant bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ is revolved about the y-axis. Find the volume.

Example 5 The region bounded by $f(x) = x^2 - 1$ and $g(x) = x + 1$ is revolved about the line $y = -2$. Find the volume.