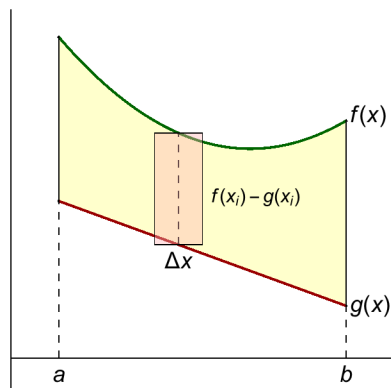


## 6.1 Area Between Two Curves

### Definition

Let  $f$  and  $g$  be continuous functions with  $f(x) \geq g(x)$  for all  $x$  on the interval  $[a, b]$ . The area bounded by the graphs of  $f$  and  $g$  between  $x = a$  and  $x = b$  is given by

$$A = \int_a^b [f(x) - g(x)] dx \quad (2)$$



One way to remember this is to think of three steps: **1.** Slice, **2.** Approximate, **3.** Integrate

**Example 1** Find the area bounded by the functions  $f(x) = x^2 - 5x + 8$  and  $g(x) = x + 3$ .

**Example 2** Find the area bounded by the functions  $f(x) = 5 - x$ ,  $g(x) = \frac{\sqrt{x}}{2}$  and the  $x$ -axis.

**Example 3** Find the area bounded by the curve  $y = \sqrt{\frac{1}{x} - 1}$  and the  $x$ -axis on the interval  $[\frac{1}{2}, 1]$

**Example 4** Find the area bounded by the graphs of  $x = y^2 - 2y$  and  $x - y - 4 = 0$ .

**Example 5** Find the area between the curves  $y = x$ , and  $y = x^2$ ;  $y = x$ , and  $y = x^3$ ;  $y = x$ , and  $y = x^n$  and take the limit as  $n \rightarrow \infty$ .

**Challenge Problem** Find the slope for the line  $f(x) = mx + 2$  such that the bounded area between  $f(x)$  and  $g(x) = x^2 - 2x + 2$  is 10.