# 5.5 The Substitution Rule

Suppose you need evaluate the indefinite integral $\int 5x\sqrt{3x^2+2}\,dx$. The antiderivative is not so easy to see. However, using a change of variable, the integrand can be made simpler. Suppose we let $u=3x^2+2$. Now, the *differential* of $u$ is $du=6x\,dx$, which means $\frac{1}{6}du=x\,dx$. We can now rewrite the integral in terms of *u,* and integrate:

$$\begin{aligned}\int 5x(3x^2+2)^{1/2}\,dx &= 5\int x(3x^2+2)^{1/2}\,dx\\ &= 5\cdot\tfrac{1}{6}\int u^{1/2}\,du\\ &= \tfrac{5}{6}\cdot\tfrac{2}{3}u^{3/2}+C\\ &= \tfrac{5}{9}(3x^2+2)^{3/2}+C\end{aligned}$$

**Note:** The last step was to back substitute to get the expression back in terms of *x*.

### The Substitution Rule

If $u=g(x)$ is a differentiable function whose range is an interval $I$ and $f$ is continuous on $I$, then

$$\int f(g(x))\,g'(x)\,dx=\int f(u)\,du$$

### Steps for Using the Substitution Rule

1. If the integrand contains a composite function $f(g(x))$, let $u=g(x)$.
2. Using $u=g(x)$, find $du=g'(x)\,dx$, and rewrite the integral in terms of $u$ and $du$ (NO $x$'s should remain.)
3. Solve the integral in terms of $u$.
4. Use $u=g(x)$ to rewrite the answer in terms of $x$.

**Example 1** Evaluate: $\int x^2(x^3+5)^9\,dx$

**Example 2** Evaluate: $\int \frac{3\cos(x)}{2\sqrt{4+\sin(x)}}\,dx$

**Example 3** Evaluate: $\int_0^5 x\sqrt{x+4}\,dx$

**Example 4** Evaluate: $\int x^5\sqrt{x^3+1}\,dx$

**Example 5** Evaluate: $\int \frac{(\ln(x)+1)\cos\left(3(\ln(x)+1)^2+5\right)}{x}\,dx$

## Symmetry

Suppose $f$ is continuous on $[-a, a]$,

a) If $f$ is an even function, i.e., $f(-x) = f(x)$, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

b) If $f$ is an odd function, i.e., $f(-x) = -f(x)$, then $\int_{-a}^{a} f(x)\,dx = 0$

**Example 6** Evaluate the integral: $\int_{-2}^{2} t^2\sin\left(t^3\right)dt$.