5.4 Indefinite Integrals and the Net Change Theorem

An indefinite integral is one in which the limits of integration are not given. That is, an **indefinite integral** is

$$\int f(x)\,dx = F(x) \quad \text{means} \quad F'(x) = f(x)$$

Example 1 Find the general indefinite integral: $\int \left(\cos(x) + x^2\right) dx$.

Table of Indefinite Integrals

Complete the table of indefinite integrals

1. $\int c\, f(x)\,dx = c \int f(x)\,dx$

2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

3. $\int x^n\,dx =$

4. $\int \frac{1}{x}\,dx =$

5. $\int e^x\,dx =$

6. $\int a^x\,dx =$

7. $\int \sin(x)\,dx =$

8. $\int \cos(x)\,dx =$

9. $\int \sec^2(x)\,dx =$

10. $\int \csc^2(x)\,dx =$

11. $\int \sec(x)\tan(x)\,dx =$

12. $\int \csc(x)\cot(x)\,dx =$

13. $\int \frac{1}{x^2+1}\,dx =$

14. $\int \frac{1}{\sqrt{1-x^2}}\,dx$

Example 2 Find the general indefinite integral: $\int \left(\frac{x^2+1}{x} + 2^x\right) dx$

Example 3 Find the general indefinite integral: $\int \frac{\sin(2x)}{\sin(x)}\,dx$

Example 4 Evaluate the definite integral and interpret the result: $\int_1^4 \left(x^2 - 3x\right) dx$.

Suppose the function $f(x) = x^2 - 3x$ in example (4) represented the rate of growth in the value of a certain stock. The area, or **net change**, represents the change in the value of the stock over that time interval. This is simply another way to reformulate **FTC2**:

The Net Change Theorem

The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Example 5 The velocity of a particle moving on the number line is given by $v(t) = t^2 - 4t + 3$. Find its net change of position, *or displacement*, on the interval $[1, 5]$, and find the total distance traveled.

Example 6 The rate the altitude of a jet is changing is given by $A'(t) = t^3 - 6t$. Find a value $c > 1$ such that the net change on the interval $[1, c]$ is zero.