5.3 The Fundamental Theorem of Calculus

Fundamental Theorem of Calculus Part 2

If f is continuous on $[a, b]$ and $F(x)$ is any antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Example 1 Evaluate: $\int_0^4 x^2\,dx$

Example 2 Evaluate: $\int_1^3 (2x + 4)\,dx$

Example 3 Evaluate: $\int_{-2}^3 x^{-4}\,dx$

Example 4 Evaluate: $\int_0^{2\pi} \sin(x)\,dx$

Since $\int_a^b f(x)\,dx$ represents the net area of $f(x)$ on the interval $[a, b]$, a function representing the net area on the interval $[a, x]$ can be represented by the **area function**

$$A(x) = \int_a^x f(t)\,dt$$

Example 5 Find the area function $A(x)$ for the function $f(t) = 4t + 2$ on the interval $[2, x]$

Example 6 What is $\frac{d}{dx}A(x)$ in example 5?

Fundamental Theorem of Calculus Part 1

Let $f(t)$ be a continuous function on $[a, b]$. The area function $A(x) = \int_a^x f(t)\,dt$ where $a \le x \le b$ is continuous on $[a, b]$ and differentiable on (a, b). The area function is such that $A'(x) = f(x)$, or

$$A'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Example 7 Find: $\frac{d}{dx}\int_\pi^x \cos(3t^2 + 4)\,dt$

Example 8 Use FTC.1 to evaluate: $\frac{d}{dx}\int_3^{x^3} \frac{3t}{\sqrt{2-\sin(t)}}\,dt$

Example 9 Find an expression for: $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt$