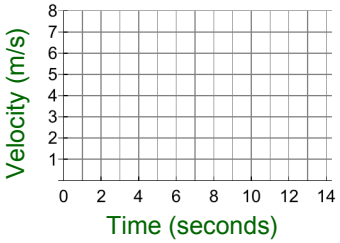
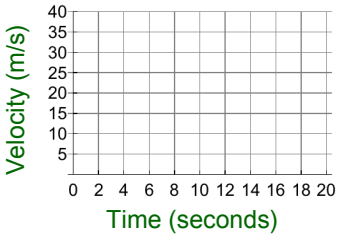
Math 152 Calculus II Notes

5.1 The Distance and Area Problems

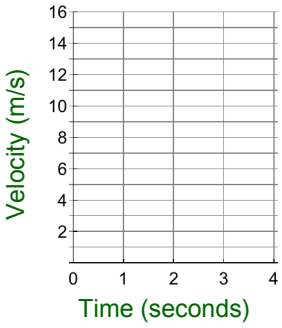
Example 1 A bicycle rider with a velocity 5 meters/sec rides for 10 seconds. How far does she travel?



Example 2 A car at rest accelerates linearly to 30 m/s taking 20 seconds. How far did the car travel on the interval [10, 20] seconds?

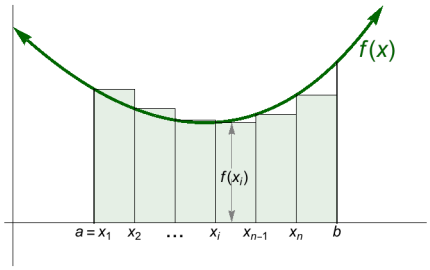


Example 3 The velocity of rocket is $v(t) = t^2$ m/s for the first 8 seconds of flight. Estimate the distance the rocket travels during the first 4 seconds of flight?



Area Under a Curve

The area under a continuous function $y = f(x)$ on the interval $[a, b]$ can by approximated with n rectangles whose width is $\Delta x = \frac{b-a}{n}$ and height is $f(x_i)$



Since each sub-rectangle has area $S_i = f(x_i)\,\Delta x$, the approximated area, R, under the curve can be written in summation notation:

$$R \approx \sum_{i=1}^{n} f(x_i)\,\Delta x = f(x_1)\,\Delta x + f(x_2)\,\Delta x + \cdots + f(x_{n-1})\,\Delta x + f(x_n)\,\Delta x \tag{1}$$

Equation (1) is referred to as a *Riemann Sum*.

Example 4 Approximate the area under $f(x) = x^2$ using the midpoint rule with $n = 4$ over the interval [0, 4].

Example 5 Approximate the area under the curve $f(x) = e^x$ between $x = 1$ and $x = 3$ with 5 sub-divisions using the left-hand endpoints.

The exact area of the region in in example (5) can be written as a limit as follows

$$\lim_{n\to\infty} \sum_{i=1}^{n} e^{1+\frac{2}{n}i} \cdot \frac{2}{n}$$

Using *Mathematica* to evaluate this limit we get

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Limit[∑_{i=1}^{n} e^(1+2/n i) 2/n, n → ∞]
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$$e\left(-1 + e^2\right)$$

or $e^3 - e \approx 17.3673$.

Example 6 Write the area under the curve $f(x) = \frac{\ln(x)}{x}$ on the interval $3 \le x \le 7$ as a limit of a Riemann Sum.