4.7 Optimization

Example 1 The sum of two numbers, x and y, is 10. Find the maximum value of $Q = 3xy + y^2$. Find the minimum value.

Example 2 A farmer wishes to build a rectangular corral with three separate pens against the side of a barn with 600 feet of fencing. What are the dimensions that will maximize the total area?

Example 3 A box is to be made from a 15 cm × 20 cm piece of paper by cutting out squares from each corner and folding up the four sides. Find the dimensions of the square to be cut out and the maximum volume of the box.

Example 4 Find a function that models the distance from the point (1, 3) to the curve $f(x) = x^2$. Graph the distance function and find the point on the curve that is closest to (1, 3).

Example 5 Find the maximum volume cylinder that can be placed inside of a right circular cone with radius 20 cm and height 60 cm.

Miscellaneous Optimization Problems

1. Find the dimensions of a cylinder with one closed end with fixed volume V that minimizes surface area.

2. A 100 cm wire is cut into two pieces. One piece is bent to make a circle, the other bent into a square.

a) Where should the wire be cut to minimize the total area of the circle and square?

b) Where should the wire be cut to maximize total area?

3. What is the area of the largest rectangle that can be inscribe under the curve of $f(x) = \sqrt{10 - x}$ in the first quadrant?

4. What is the ratio of $\frac{\text{height}}{\text{radius}}$ of the cylinder that can be inscribed in a sphere of radius k such that the cylinder has maximum

a) surface area? **b)** volume? **c)** are these the same?

5. Suppose a dog can run 15 feet per second on the sand and swim 5 feet per second in the water. Chuck and his dog Lily are playing fetch at the beach with a stick. Where should Lily enter the water if a stick is thrown 80 feet down the beach and 40 feet out into the water? What is the minimum time?

6. A circular cylinder is constructed with material that costs \$0.005 per square centimeter for the top and bottom, and \$0.002 per square centimeter for the sides. Find the dimensions of the cylinder that will minimize the cost if the volume of the cylinder is to be 500ml; and find the minimum cost.

7. A clock has an 8 inch minute hand and a 5 inch hour hand. Find the angle between the hands that maximizes the area of the triangle formed by the hands. (What time after 12:00 does this happen?)

8. The seating in a movie theater is slanted at a 20° from the horizontal. The movie screen is place 10 feet above the floor and is 25 feet high. Where should a person sit to maximize their viewing angle? (First, for a simpler problem, assume the seats are not on an angle, but rather on the level.) Might need *Mathematica* to solve this one.