4.5 Sophisticated Graphing

Concepts from Pre-Calculus

1. Domain and Range of a Function

1.1. Domain: all possible x-values

1.2. Range: all possible y-values (often difficult; usually need some later techniques)

2. Intercepts

2.1. x-intercepts: whenever $f(x) = 0$

2.2. y-intercepts: $y = f(0)$

3. Tool-kit functions

Knowing the general shape of basic functions: Linear, quadratic, power functions x^n, root functions $x^{1/n}$, reciprocal $\frac{1}{x}$, sine, cosine, etc.

Concepts from Calculus (and some from Pre - Calculus)

1. Asymptotes and Holes

1.1. A reduced rational function $f(x) = \frac{P(x)}{Q(x)}$ has vertical asymptotes $x = c$ whenever $Q(c) = 0$. Use $\lim_{x \to c^-} f(x)$ and $\lim_{x \to c^+} f(x)$ to find the behavior of the vertical asymptotes.

1.2. A reduced rational function $f(x) = \frac{P(x)}{Q(x)}$ has horizontal asymptotes $y = L$ if $\lim_{x \to -\infty} f(x) = L$ or $\lim_{x \to +\infty} f(x) = L$.

Example: $f(x) = \frac{3x^2+2x-5}{2x^2+x-1}$

1.3. A "hole" can occur if $P(x)$ and $Q(x)$ have a common factor, e.g. when $f(x) = \frac{P(x)}{Q(x)}$ is not reduced. Example: $f(x) = \frac{x+2}{x^2-4}$.

1.4. Slant asymptotes occur when the degree of $P(x)$ is greater that $Q(x)$. Example $f(x) = \frac{x^2+5x-3}{x-1}$

2. Critical Numbers and Local Extrema (possible extrema)

2.1. $f'(x) = 0$, or

2.2. $f'(x)$ is undefined

Use either the first or second derivative test to determine the type of local extrema.

3. Intervals of Increasing or Decreasing

3.1. Intervals on which $f'(x) > 0$ are increasing

3.2. Intervals on which $f'(x) < 0$ are decreasing

4. Inflection Points (possible)

4.1. $f''(x) = 0$, or

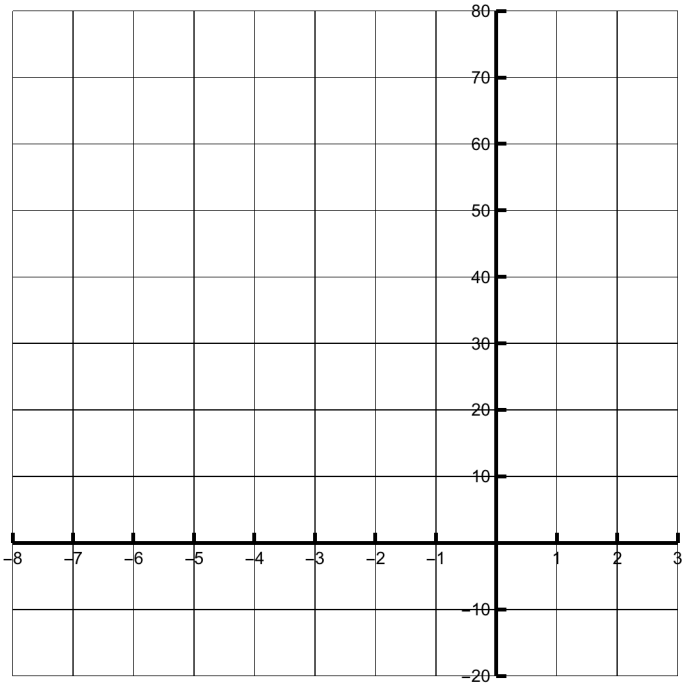
4.2. $f''(x)$ is undefined

5. Intervals of Concavity

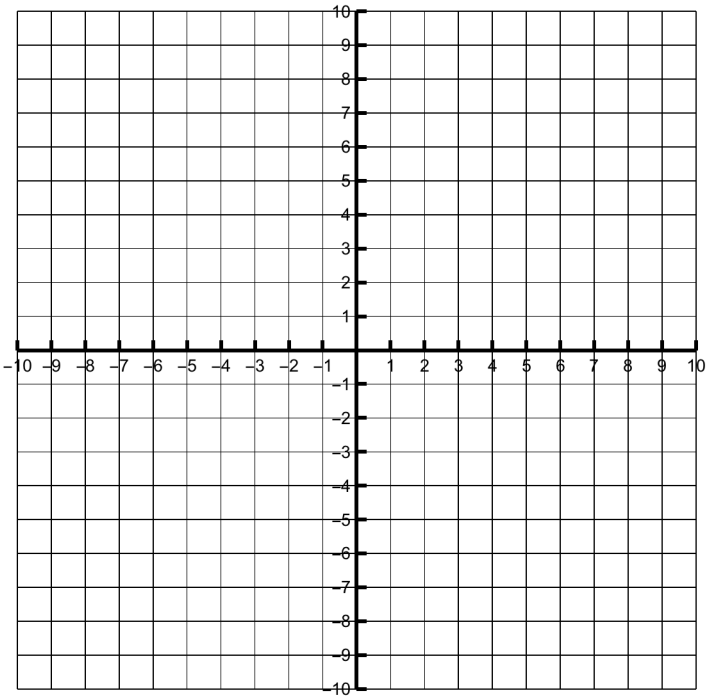
5.1. Intervals on which $f''(x) > 0$ are concave up

5.2. Intervals on which $f''(x) < 0$ are concave down

Example 1 Make an accurate sketch of $f(x) = x^3 + 6x^2 + 9x + 54$ by indicating x-intercepts, y-intercepts, asymptotes, critical values, inflection points, etc.



Example 2 Make an accurate sketch of the rational function $f(x) = \frac{x^2 - 2x + 4}{x - 2}$ indicating intercepts, asymptotes, local extrema, etc.



Example 3 Analyze the function $f(x) = \frac{\sqrt{x}\,(x-5)^2}{4}$