4.4 Indeterminate Forms and L'hospital's Rule

Back in Chapter 2 we evaluated various limits that were indeterminate, e.g, of the forms $\frac{0}{0}$, $\frac{\infty}{\infty}$. Other indeterminate forms are $\infty - \infty$, $0 \cdot \infty$, 0^0, ∞^0, 1^∞, etc. The method was to simplify the expression algebraically and then analyze. For the indeterminate form $\lim\limits_{x\to a} \frac{f(x)}{g(x)} \to \frac{0}{0}$ (or $\to \frac{\infty}{\infty}$) the existence of the limit depends on the **rate** the numerator and denominator each go to 0 (or to ∞). For the indeterminate forms the limit depends on which piece of the expression "wins the race", or is there a compromise. This idea leads to L'hospital's Rule for indeterminate limits.

L'hospital's Rule

If $\lim_{x\to a} \frac{f(x)}{g(x)}$ is of the indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Note: this **not** the quotient rule.

Example 1 Evaluate the following limit algebraically, and then using L'hospital's Rule: $\lim\limits_{x\to 3} \frac{x^2-3x}{x^2+2x-15}$.

Example 2 Evaluate: $\lim\limits_{x\to 0} \frac{\cos(x)-1}{x}$

Example 3 Evaluate: $\lim\limits_{x\to 1} \frac{\ln(x)}{x-1}$

Example 4 Evaluate: $\lim\limits_{x\to \infty} \frac{e^{2x}}{x}$

Example 5 Evaluate: $\lim\limits_{x\to 0} \frac{\sin(x)-x}{x^3}$

Indeterminate Products

Example 6 Evaluate: $\lim\limits_{x\to\infty} x\cdot\tan\left(\frac{1}{x}\right)$

Indeterminate Powers

Example 7 Evaluate: $\lim\limits_{x\to 0^+} (\sin(x))^x$

Example 8 Evaluate: $\lim\limits_{x\to\infty} \left(1+\frac{1}{x}\right)^x$

Indeterminate Differences

Example 9 $\lim\limits_{x\to 1^+} \left(\frac{x}{x-1}-\frac{1}{\ln(x)}\right)$