4.2 Rolle's Theorem and The Mean Value Theorem

Rolle's Theorem

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) such that $f'(c) = 0$.

Proof

Example 1 Demonstrate Rolle's Theorem by finding the x-intercepts of $f(x) = x^2 - 3x + 2$ and showing that $f'(x) = 0$ for some x between the x-intercepts.

Example 2 Use the Intermediate Value Theorem and Rolle's Theorem to prove the equation $x^3 + 2x - 2 = 0$ has only one solution.

The Mean Value Theorem

If f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b), then there is at least one number c in (a, b) where

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

or, equivalently, where $f(b) - f(a) = f'(c)(b - a)$.

PROOF

Example 3 Find the number c guaranteed by the Mean Value Theorem for $f(x) = 2\sqrt{x}$ on $[1, 4]$.

Example 4 A car is at a toll booth and gets a time-stamped receipt. Eighteen minutes later and 20 miles down the road at another toll booth its speed is clocked at 50 mph, yet a policeman hands him a ticket for going over 60 mph. Why?