

## 4.10 Antiderivatives

### Notation for Antiderivatives

The function  $f(x)$  has an anti-derivative  $F(x) + C$  if  $\frac{d}{dx}[F(x) + C] = f(x)$  for all  $x$ . The notation for the anti-derivative is

$$\int f(x) dx = F(x) + C$$

and is called an *indefinite integral*.

### Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

**Example 1** Find the indefinite integrals

a.  $\int x^5 dx$

b.  $\int \sqrt{x} dx$

### Constant Factor Property

$$\int c \cdot f(x) dx = c \int f(x) dx$$

**Example 2** Find the indefinite integrals

a.  $\int \frac{5}{x\sqrt[4]{x}} dx$

b.  $\int \frac{6}{\sqrt{1-x^2}} dx$

### Natural Logarithm Rule

$$\int \frac{1}{x} dx = \ln |x| + C$$

### Natural Exponential Rule

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

**Example 3** Evaluate  $\int e^{-2x} dx$

## Sum and Difference Property

$$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

**Example 4** Find the indefinite integrals:

a.  $\int (3e^x + 7 \sec^2(x)) dx$

b.  $\int \frac{3xe^{5x} - 4\sqrt{x} + 5}{3x} dx$

c.  $\int \frac{4+2x}{1+x^2} dx$

d.  $\int 3(4x+1)^5 dx$

## Initial Conditions and the Constant of Integration

**Example 5** Suppose the derivative of a function is  $f'(x) = 2x - 5e^x$  and the function passes through the point  $(0, 2)$ , that is  $f(0) = 2$ . Find the function.

**Example 6** The velocity of a particle traveling on the number line at any time  $t$  is given by  $v(t) = -4t + 8$ . If the particle is at position 20 at time  $t = 2$  find its location at  $t = 3$ . Recall  $s'(t) = v(t)$ .