4.1 Maximum and Minimum Values

Local Extrema and Absolute Extrema

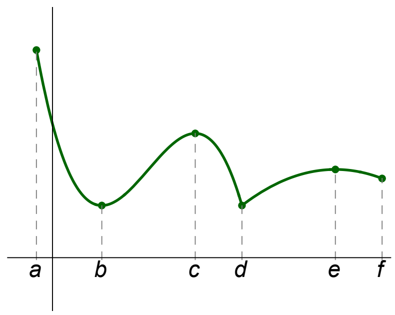
Definition

A function f has a **local** (or relative) **maximum** at $x = c$ if $f(c) \geq f(x)$ when x is near c. Similarly, f has a **local** (or relative) **minimum** at $x = c$ if $f(c) \leq f(x)$ when x is near c.

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f attains an **absolute** (or global) **maximum** value $f(c)$ and an **absolute** (or global) **minimum** value $f(d)$ at some numbers c and d in $[a, b]$.

Example 1 Identify the local and absolute extrema for the function below.



Example 2 For Theorem 1, why is continuity and "closed" necessary conditions?

Fermat's Theorem

If f has a local maximum or minimum at $x = c$, and if $f'(c)$ exists, then $f'(c) = 0$.

Definition

A **critical number** is a value $x = c$ such that either $f'(c) = 0$ or $f'(c)$ is undefined. Note, if a function f has a local extrema $f(c)$, then $x = c$ is a critical number of f.

Example 3 Find the critical numbers for $f(x) = x^{2/3}(3 - x)$.

Example 4 Find the critical numbers for $f(x) = \sin(x) + \cos^2(x)$.

Example 5 A particle moving along the number line has position $s(t) = 3\,t\,e^{-0.1\,t}$. Find the position of the particle when its velocity is at a local minimum or maximum. Which is it? When is it at a global extrema?

Example 6 The local max or min value for a quadratic $f(x) = a\,x^2 + b\,x + c$ is always at its vertex. Find the critical number for the general quadratic.

Example 7 Find the absolute extrema for $f(x) = \frac{\log_5(x)}{x}$ on the interval [1, 5].