3.9 Derivatives of Hyperbolic Functions

- Consider the circular functions sine, cosine, and tangent. They are called circular because their values are calculated using a unit circle whose equation is $x^2 + y^2 = 1$. Consider the unit circle and terminal point $p = (\cos(\theta), \sin(\theta))$ in Figure 18:

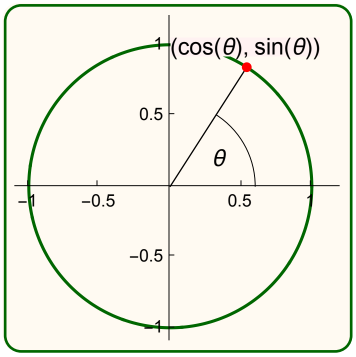

Figure 3.9.2: Circular Functions

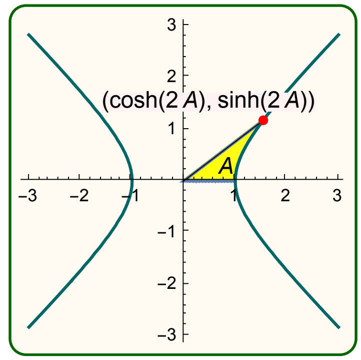

Figure 3.9.3: Hyperbolic Functions

Another way to define the coordinates of the terminal point is to use the area of the *sector*. Since the area of the sector is $A = \frac{\theta}{2}$, we have $\theta = 2A$. This gives a new definition of sine and cosine

$$x = \cos(2A) \quad \text{and} \quad y = \sin(2A)$$

- Hyperbolic functions are defined similarly using a specified area with the hyperbola $x^2 - y^2 = 1$, Figure 19. The specified point on the hyperbola is given by $p = (\cosh(2A), \sinh(2A))$.

Graphs of the Hyperbolic Functions

The graphs of the hyperbolic functions are

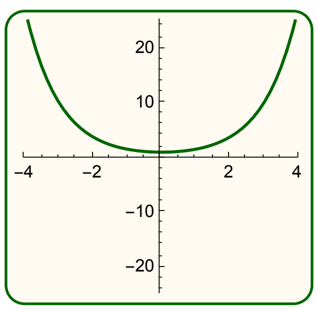

Figure 3.9.5: $f(x) = \cosh(x)$

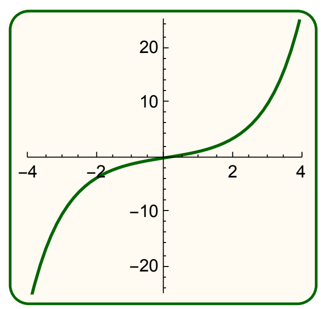

Figure 3.9.6: $f(x) = \sinh(x)$

Exponential Form of Hyperbolic Functions

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}$$

Example 1 Graph the exponential form for $\cosh(x)$ and $\sinh(x)$ on your calculator.

Example 2 Using the exponential definitions of sinh and cosh, find (a) $\frac{d}{dx}[\sinh(x)]$, and (b) $\frac{d}{dx}[\cosh(x)]$.

Example 3 Find the exponential expression for $\tanh(x)$.

Example 4 Find $\frac{d}{dx}[\tanh(x)]$

Example 5 The shape of a cable hanging under its own mass is given by $f(x) = a\cosh\left(\frac{x}{a}\right)$ and is called a *catenary curve*. Graph the catenary curve $f(x) = 5\cosh\left(\frac{x}{5}\right)$ for $-10 \le x \le 10$ and $0 \le y \le 20$, and find the slope of the tangent line at $x = 5$.