

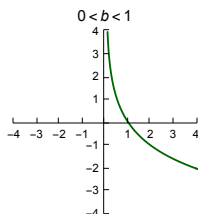
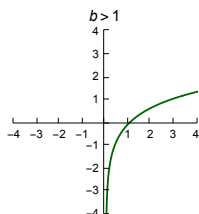
3.8 Derivatives of Logarithm Functions

Objectives: Find derivatives of logarithmic functions; use logarithmic differentiation to find derivatives.

Review of Properties of Logarithms

- $x = b^y \Rightarrow \log_b(x) = y$
- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
- $\log_b(x^n) = n \log_b(x)$
- $\log_b(1) = 0$
- $\log_b(b) = 1$
- $\log_b(b^x) = x$
- $b^{\log_b(x)} = x$
- $\log_e(x) = \ln(x)$ and $\log_{10}(x) = \log(x)$
- $\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$ or $\log_b(a) = \frac{\ln(a)}{\ln(b)} = \frac{\log(a)}{\log(b)}$

Graphs of Logarithm Functions: $f(x) = \log_b(x)$:



Theorem 1 For any positive number x (e.g. $x > 0$)

$$\frac{d}{dx}[\ln(x)] =$$

Proof

Example 1 Find the derivative of $f(x) = x^4 \ln(x)$

Example 2 Find the derivative of $g(x) = \sqrt{\ln(x) + x^2}$

Theorem 2 Chain Rule with the Natural Logarithm

$$\frac{d}{dx}[\ln(f(x))] = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)} \quad \text{or} \quad \frac{d}{dx}[\ln(u)] = \frac{1}{u} \cdot \frac{du}{dx}$$

Example 3 Find $f''(x)$ for $f(x) = \ln(\sin(2x))$

Example 4 Find the derivative of $h(x) = 3 \ln(x^3 - 4x + 5)$

Example 5 Find the derivative of $G(x) = \ln\left(\frac{2x^2+7x-2}{x^2-4x}\right)$

Logarithmic Differentiation

Sometimes taking the logarithm of each side of an equation $y = f(x)$, differentiating, and then solving for y' can be easier than traditional differentiating.

Example 6 Find the derivative of $y = \frac{x^3 \sqrt{x+1}}{\sin(4x)}$

Example 7 Find the derivative of the “super exponential function” $f(x) = x^x$.

Example 8 Use the change of base formula to find the derivative of $f(x) = \log_b(x)$.

Challenge Find $\frac{dy}{dx}$ for the implicit function $2xy^2 + \log_3(xy) + 2^{x-y} = 3$ at the point $(1, 1)$.