3.7 Higher Order Derivatives

Objectives: Find higher order derivatives; solve application problems using higher order derivatives. FInd higher order derivatives of implicit functions.

$$\text{1 st Derivative : } y' = \frac{dy}{dx} = f'(x) = \frac{d}{dx}[f(x)]$$
$$\text{2 nd Derivative : } y'' = \frac{d^2y}{dx^2} = f''(x) = \frac{d^2}{dx^2}[f(x)]$$
$$\text{3 rd Derivative : } y''' = \frac{d^3y}{dx^3} = f'''(x) = \frac{d^3}{dx^3}[f(x)]$$
$$\text{4 th Derivative : } y^{(4)} = \frac{d^4y}{dx^4} = f^{(4)}(x) = \frac{d^4}{dx^4}[f(x)]$$
$$\vdots$$

Example 1 Find the second derivative of $f(x) = x^3 - \sqrt{x} + \frac{2}{x} + 7x - 2$

Physical Application

If $s(t)$ represents the location of an object at time t, then the rate of change in the location (i.e. distance) is velocity:

$$s'(t) = \frac{ds}{dt} = v(t)$$

The rate of change in velocity is acceleration:

$$s''(t) = \frac{d^2s}{dt^2} = v'(t) = a(t)$$

Example 2 The position of a particle from its starting point is $s(t) = t^3 - 9t^2 + 70$. Find its position and velocity when its acceleration is 0.

Example 3 The value of a share of stock is given by $V(t) = \frac{30t}{4+3t}$ t years after going public. Find the rate of growth in the value after 4 years. Is the rate of growth increasing or decreasing?

Example 4 Find $f''(1)$ if $f(x) = \left(x^2 - 3x\right)^4$

Example 5 Given the implicit function $y^2 + xy = 8$ find an expression for $\frac{d^2y}{dx^2}$ in terms of x and y only (i.e., not y').

Example 6 Find $f''(x)$ for $f(x) = \tan^{-1}(x)$ and graph all three functions, f, f', f''. (See if you can find f''').