3.6 Implicit Differentiation

Objectives: Use implicit differentiation to find the dy/dx of implicit functions; derive formulas for the derivative of inverse trigonometric functions.

An explicit function is a function that can be written as a function in a single variable, e.g., $f(x) = x^2 + 5x - 5$. An implicit function is a function that is written in terms of 2 or more variables, such as $2x^2y^2 - 5y + 2 = x^2$. How can you find $\frac{dy}{dx}$?

Implicit Differentiation

Assume y is a function of x. Then, taking the derivative of y is: $\frac{d}{dx}[y] = y' = \frac{dy}{dx}$.

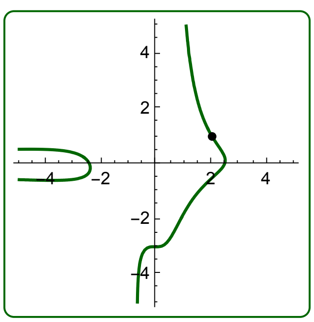
Power Rule

If y is a function of x, i.e., $y = f(x)$, then

$$\frac{d}{dx}[y^n] = n\,y^{n-1} \cdot \frac{dy}{dx}, \quad \text{or}$$
$$= n\,y^{n-1} \cdot y'$$

Example 1 Find $\frac{dy}{dx}$ for the implicit function $3x^2 + y = 4y^3$ at the point (1, 1).

Example 2 Find $\frac{dy}{dx}$ for the implicit function $2x^2 + x^3y^2 - 4y = 12$, and find the value of $\frac{dy}{dx}$ at (2, 1)



Example 3 Given $y\sin(x^2) = x\sin^2(y)$ find a) $\frac{dy}{dx}$ and b) $\frac{dx}{dy}$.

Orthogonal Trajectories

Two curves are **orthogonal** if they intersect at right angles.

Example 4 Show that the circle $x^2 + y^2 = 25$ and the parabola $y = \frac{2}{9}x^2 + 2$ are orthogonal. Recall, two slopes are perpendicular if $m_1 m_2 = -1$.

Example 5 Show that the given families of curves are orthogonal trajectories of each other.

$y = a x^3$ and $x^2 + 3y^2 = b$

Example 6 Find a quadratic that is orthogonal to $y = x^2$ at the points $(\pm 2, 4)$.

Derivatives of the Inverse Trigonometric Functions

Example 7 Use implicit differentiation to find a formula for $f(x) = \sin^{-1}(x)$.

Example 8 Use implicit differentiation to complete the table:

$\frac{d}{dx}\sin^{-1}(x) =$	$\frac{d}{dx}\cos^{-1}(x) =$	$\frac{d}{dx}\tan^{-1}(x) =$
$\frac{d}{dx}\csc^{-1}(x) =$	$\frac{d}{dx}\sec^{-1}(x) =$	$\frac{d}{dx}\cot^{-1}(x) =$