3.4 Derivatives of Trigonometric Functions

Objectives: Find the derivative of sine, cosine, and tangent functions; find the derivative cosecant, secant, and cotangent function. Evaluate limits involving sine and cosine.

Derivatives of Sin, Cos, Tan

$\frac{d}{dx}\sin(x) =$	$\frac{d}{dx}\cos(x) =$	$\frac{d}{dx}\tan(x) =$

Proof for $\frac{d}{dx}\sin(x)$

Example 1 Use the *quotient rule* to derive the derivative of $\tan(x)$.

Example 2 Find the derivative of $f(x) = x^2\cos(x)$.

Example 3 Use the double angle formula to find $\frac{d}{dx}[\sin(2x)]$.

Example 4 Find the points on the curve of $y = \frac{\cos(x)}{2+\sin(x)}$ where the tangent line is horizontal.

Example 5 Derive the derivative for $\sec(x)$.

Example 6 Derive the derivative for $\csc(x)$.

Example 7 Derive the derivative for $\cot(x)$.

Summary

$\frac{d}{dx}\sin(x) =$	$\frac{d}{dx}\cos(x) =$	$\frac{d}{dx}\tan(x) =$
$\frac{d}{dx}\csc(x) =$	$\frac{d}{dx}\sec(x) =$	$\frac{d}{dx}\cot(x) =$

Example 8 Calculate $\lim_{x\to 0}\frac{\sin(8x)}{3x}$.

Example 9 Calculate $\lim_{x\to 0}\frac{\sin(6x)}{\sin(4x)}$.

Challenge Given the parabolas $f(x) = x^2$ and $g(x) = -x^2 + 6x - 18$, how much does g need to be shifted vertically so that the two parabolas are tangent.