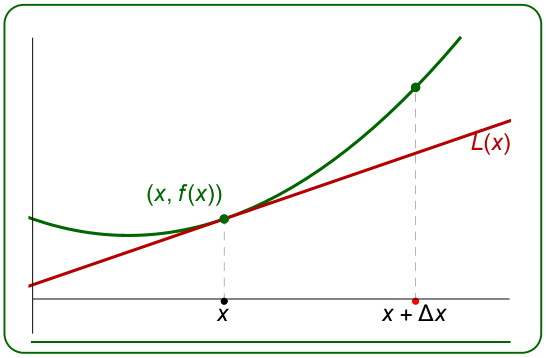
3.11 Linear Approximations and Differentials

Objectives: Use differentials to approximate Δy; use differentials to approximate $f(x + \Delta x)$; find the linearization of a function $f(x)$.

Differentials

Differentials will play several roles throughout calculus, but here we'll use them mainly for approximations. Consider the tangent line $L(x)$ for the function $f(x)$ below:



Differentials

The *actual* change in the function from x to $x + \Delta x$ is Δy. The approximate change in y using the tangent line is dy. The values dx and dy are called **differentials**:

$dx =$	$dy =$

Example 1 Find the differential dy for: a) $y = \sin\left(x^2 + 5x - 8\right)$ b) $y = \sqrt{x^2 + 1}$

Approximations

We can use differentials to approximate function values. First note that $f(x + \Delta x) = f(x) + \Delta y$, or $f(x + \Delta x) \approx f(x) + dy$, or finally:

$$f(x + \Delta x) \approx$$

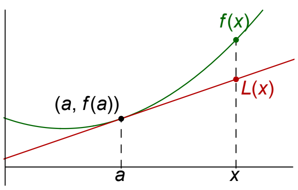
Example 2 Suppose you need good approximations to $\sqrt{3.85}$ and $\sqrt{9.4}$ but don't have a calculator. Use differentials to approximate these values.

Example 3 A soap bubble expands from a diameter of 40 mm to 50 mm. Use differentials to approximate the change in the surface area.

Example 4 The edge of a cube is measured to be 20.5 inches with an error of ± 0.25 inches. Use differentials to estimate the maximum error in volume of the cube. Calculate the *relative error* of the estimate: $E = \frac{\Delta V}{V}$.

Linear Approximation

Example 5 Show that the **linearization** of a function $f(x)$ at $x = a$ is given by $\boxed{L(x) = f(a) + f'(a)(x - a)}$.



Example 6 Find the linearization of $f(x) = x^2 \sin(x) =$ at $x = \frac{\pi}{2}$. Plot the function and the linearization on the interval $[0, \pi]$.