3.10 Related Rates

Objective: Solve application problems involving related rates.

Rates of change are usually expressed as the change in a quantity with respect to the change in time $\frac{df}{dt}$, e.g. $\frac{\text{miles}}{\text{hour}}$, $\frac{\text{cm}^3}{\text{sec}}$, etc. Consider a spherical balloon being blown up by increasing its volume. Not only is the volume changing, but so is the radius, and the surface area. And, each is changing at a different rate related by their geometrical equations, $V = \frac{4}{3}\pi r^3$ and $SA = 4\pi r^2$.

Example 1 Assume two variables x and y are both a function of t, i.e. $x(t)$ and $y(t)$, and that $y^2 + 1 = x + 2xy$. Find $\frac{dy}{dt}$ when $x = 1$, $y = 2$, and $\frac{dx}{dt} = 3$

Example 2 The base of a 10-foot ladder leaning against a wall is being pulled from the wall at a rate of 1.0 feet per second. Find the rate of the top of the ladder when

- **a.** The top is 8 feet above the ground.
- **b.** When the base is 9 feet from the wall.
- **c.** When the ladder hits the ground.

Example 3 Superman is blowing a spherical bubble whose volume is increasing at a rate of 30 cm^3/s. Find the rate the radius is increasing when the volume is 1000 cm^3.

Example 4 A light house is positioned 1320 feet off-shore and is rotating 2 revolutions per minute. How fast does Superman need to run to keep up with the beam of light when he is 2 miles down the beach from a point perpendicular to the lighthouse? Convert the speed to miles per hour.

Example 5 A right circular cylinder has an initial radius of 4 cm and an initial height of 12 cm. The radius of the cylinder is increasing at a rate of 4 cm/min, and the height is decreasing at a rate of 3 cm/min.

- **a.** Is the volume increasing or decreasing when the height is 6 cm and the radius is 12?
- **b.** Is it increasing or decreasing after 3 minutes?
- **c.** At what time is $dv/dt = 0$?

Example 6 A small boat is being pulled towards an 8-foot high dock. The rope is being pulled in at a rate of 2 feet per second.

- **a.** Find the velocity of the boat when the length of the rope is 25 feet.
- **b.** Find the velocity when the boat is 10 feet from the dock.
- **c.** Find the velocity when the boat is touching the dock, i.e., the length of the rope is 8 feet.

Example 7 James Bond is 6-feet tall and tied to a pole and is being covered by a pile of sand that is increasing at a rate of 20 cubic feet per minute. The sand pile always has a base diameter 8 times the height.

- **a**. How fast is the height of the pile increasing when the pile is 4 feet deep?
- **b.** How fast is the height of the pile increasing when his head gets covered?
- **c.** How long does James have until he's covered?