2.9 The Derivative as a Function

Objective: Use the limit definition to find the derivative of a function $f(x)$.

Using the limit of the difference quotient given in the previous sections, we can find the limit at a general x value instead of a fixed value a. This gives a *derivative function* instead of a derivative at a fixed point.

Definition

The derivative of $f(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

Other Notation

The derivative of the function $y = f(x)$ has multiple representations. Below are a few:

$$y' = f'(x) = D_x[f] = \frac{dy}{dx} = f_x(x) = \frac{d}{dx} f(x) = \dot{y}. \qquad \left(\text{Leibniz: } \frac{dy}{dx},\ \text{Lagrange: } f'(x),\ \text{Newton: } \dot{y},\ \text{Euler: } D_x f(x)\right).$$

Example 1 Find the derivative of $f(x) = x^3 - 4x + 5$. Plot $f(x)$ and $f'(x)$ together.

Example 2 Find $f'(x)$ for $f(x) = \sqrt{x+2}$. Plot $f(x)$ and $f'(x)$ together.

Identifying a Function and It's Derivative

Copyright © 2015 CStevens

Example 3 Find $f'(x)$ for $f(x) = \frac{x+3}{x-1}$.

Definition

A function is differentiable at a if $f'(a)$ exists. It is differentiable on an open interval (a, b), (where a can be $-\infty$, and/or b can be ∞), if it is differentiable at every number in the interval.

Example 4 Where is the function $f(x) = |x|$ differentiable?

Continuity and differentiability of a function are desired properties. The following theorem relates these two characteristics of a function.

Theorem

If f is differentiable at a then f is continuous at a.

Ways a Function is Not Differentiable at a Point *a*

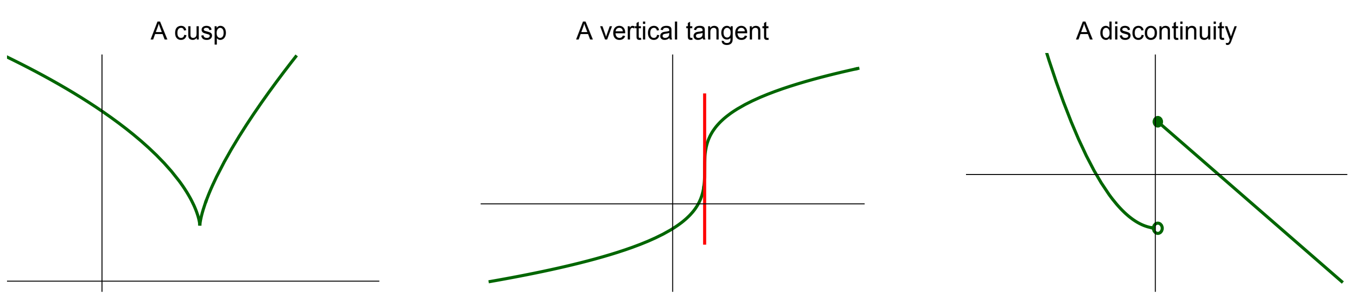

Figure 2.9.4: Non–Differentiability

Copyright © 2015 CStevens