2.8 The Derivative

Objective: Use the limit definition to find the derivative at $x = a$. Find equations of tangent lines to curves.

Definition

The derivative of a function $y = f(x)$ at a number a is defined as

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists. If $f'(a)$ exists, then we say f is differentiable at a.

Note: the above definition can also be written as $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$.

Example 1 Given the function $f(x) = x^2 - 2x + 2$ find $f'(3)$ and $f'(-2)$.

Example 2 Find $f'(a)$ for $f(x) = -x^3 + 12x + 2$, and find a where the tangent slope is 0, that is where $f'(a) = 0$.

Example 3 Find $f'(a)$ for $f(x) = \frac{2x+2}{x-1}$ and $f'(5)$. Verify by graphing the function and tangent line.

Example 4 Find the equation of the tangent line to the function $f(x) = \sqrt{x+3}$ when $x = 4$.

Example 5 Find the equation of the line tangent to $f(x) = e^x$ when $x = 1$.