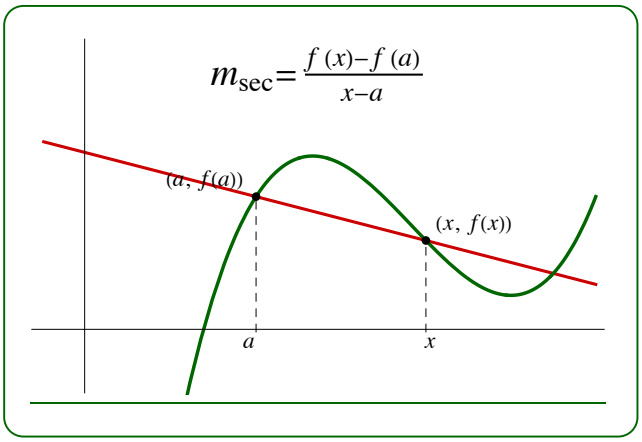
2.7 Tangents, Velocities, and Other Rates of Change

Objectives: Find the slope of the tangent line to a curve; use the difference quotient to find the tangent slope; find other rates of changes.

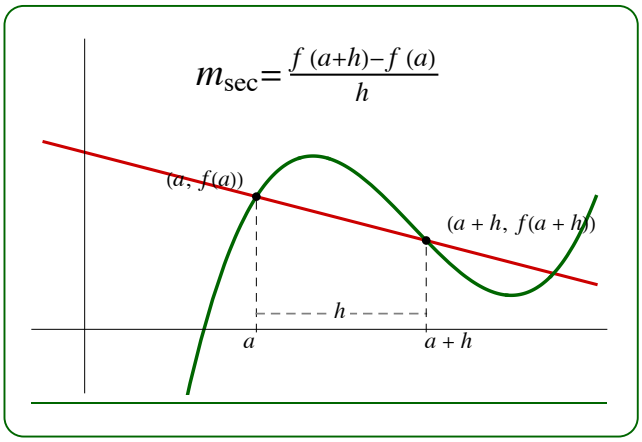
In section 2.1 we made educated guesses in finding the slope of tangent lines and velocities. An important application of calculus is the ability to find the tangent line (or linear approximation) of a function, and instantaneous velocities easily. Being able to calculate various limits using the limit laws allows us to to easily calculate tangent slopes, instantaneous velocities, and other rates of changes.

Consider the following two ways of finding the tangent slope of a line to a function using the *difference quotients*:

Method I



Method II



Example 1 Find the equation of the line tangent to $y = \frac{4}{x}$ at $a = 2$ using method I.

Example 2 A rock has position $s(t) = -16t^2 + 200$ (in feet) t seconds after being dropped from a height of 200 feet. Find the instantaneous velocity after 3 seconds using method II.

Example 3 The radius of an oil spill is expanding at a rate such that after t minutes the radius is $r = 4\sqrt{t}$ meters. Find the rate the radius is changing after 30 minutes.

Example 4 Find the point on the curve of $y = 2x^2 - 3x + 2$ where the tangent has a slope of $m = 5$. Find the tangent line and graph it and the function.

Example 5 Find the equation of the tangent line to the curve $y = \frac{x+2}{x-1}$ when $x = 2$.