2.3 Calculating Limits with Limit Laws

Objective: Use limit laws to algebraically evaluate limits. Use the squeeze theorem to evaluate applicable limits.

Limit Laws

Suppose c is a constant and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exists.

1. $\lim_{x\to a} c = c$

2. $\lim_{x\to a} x = a$

3. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a} [f(x) + g(x)] = L + M$

4. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a} [f(x) - g(x)] = L - M$

5. $\lim_{x\to a} (c \cdot f(x)) = c \lim_{x\to a} f(x)$

6. $\lim_{x\to a} [f(x) \cdot g(x)] = L \cdot M$

7. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

8. $\lim_{x\to a} (x^n) = \left(\lim_{x\to a} x\right)^n$

Example 1 Find $\lim_{x\to 2} \left(x^2 - 3x + 5\right)$

Direct Substitution Property

If f is a polynomial or rational function and a is the domain of f, then

$$\lim_{x\to a} f(x) = f(a)$$

Example 2 Find the limit if it exists: $\lim_{x\to -2} \frac{x^2-4x-5}{x^2+4}$.

Example 3 Find the limit if it exists: $\lim_{x\to 1} \frac{x^2-1}{x-1}$.

9. $\lim_{x\to a}\left(f(x)^n\right)=\left(\lim_{x\to a}f(x)\right)^n$

10. $\lim_{x\to a}\left(\sqrt[n]{f(x)}\right)=\sqrt[n]{\lim_{x\to a}f(x)}$, if n is even we assume $f(x)\geq 0$.

Example 4 Find the limit if it exists: $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$

Example 5 Determine the value c such that the limit exists at $x\to 2$: $f(x)=\begin{cases}\frac{\sqrt{x^2+5}-3}{x-2} & \text{if } x<2\\ \frac{x^3-8}{x-2}+c & \text{if } x>2\end{cases}$

The Squeeze Theorem

If $f(x)\leq g(x)\leq h(x)$ when x is near a (except possibly at a), and $\lim_{x\to a}f(x)=\lim_{x\to a}h(x)=L$, then $\lim_{x\to a}g(x)=L$.

Example 6 Use the Squeeze Theorem to evaluate $\lim_{x\to 0}\left(x^2\cos\left(\frac{2\pi}{x}\right)\right)$.

Example 7 Section 2.3, problem 60