2.2 The Limit of a Function

Objectives: To numerically estimate the value of a limit; determine the limit and value of a function given a graph of the function. Determine the value of one-sided limits.

Example 1 Use a table of values to investigate the value of $f(x) = \frac{2^x - 1}{x}$ as x gets close to 0.

Definition

We write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$ as x approaches a equals L", if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a.

Example 2 Guess the value of $\lim_{x \to 0} \frac{\sin(x)}{x}$.

Example 3 Guess the value of $\lim_{x \to 0} \cos\left(\frac{2\pi}{x}\right)$.

Example 4 Guess the value of $\lim_{x \to 1} \frac{|x^2 - 1|}{x - 1}$.

One Sided Limits

Definition

A **left-hand limit** is a limit where x approaches a from the left-side, or $x < a$, denoted: $\lim_{x\to a^-} f(x)$.

A **right-hand limit** is limit where x approaches a from the right-side, or $x > a$, denoted: $\lim_{x\to a^+} f(x)$.

Existence of a Limit

A limit exists at $x = a$, meaning $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L$, $\lim_{x\to a^+} f(x) = L$ and L is a finite number.

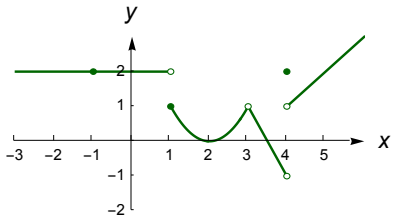
Example 5 Given $f(x) = [\![x]\!]$ determine the existence of:

(a) $\lim_{x\to 3} f(x)$ (b) $\lim_{x\to 2.5} f(x)$

Example 6 Determine the existence of: $\lim_{x\to 3} \frac{1}{(x-3)^2}$.

Example 7 Find $\lim_{x\to 0^+} \ln(x)$

Example 8 For the function f below, find the following:



a. $\lim_{x\to -1} f(x)$_____ b. $f(-1)$_____ c. $\lim_{x\to 1} f(x)$_____

d. $f(3)$_____ e. $\lim_{x\to 3} f(x)$_____ f. $f(4)$_____

f. $\lim_{x\to 1^+} f(x)$_____ g. $\lim_{x\to 1^-} f(x)$_____ h. $\lim_{x\to 4} f(x)$_____