Math 151 Calculus I In-Class Notes

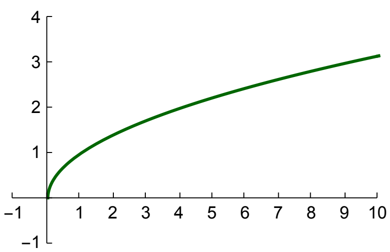
2.1 Tangent and Velocity Problems

Objectives: Discover the idea of the *limit* in finding the slope of a tangent line and the instantaneous velocity of an object.

The Tangent Line Problem

From Pre-Calc we know the slope of a line is "rise over run", or, the change in y divided by the change in x, or $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, where Δy means the *change in y*, and Δx is the *change in x*. We can find the slope of a **secant line** passing through any two points on the graph of a function.

Example 1 For the function $f(x) = \sqrt{x}$, find the slope of the secant line using $x_1 = 2$ and $x_2 = 6$. Calculate several secant slopes as x_2 gets closer to x_1, e.g., $x_2 = 4$, $x_2 = 3$, etc.



Now suppose we want to find the slope of the line **tangent** to a curve at a single point. This is not possible using the slope formula since two points are required.

Example 2 Estimate the slope of the line tangent to $f(x) = \sqrt{x}$ at $a = 2$, by calculating several secant slopes between a and x letting $x \to a$.

Definition

We say the **slope of the tangent line** at $x = a$ is the **limit** of the secant slopes as x approaches a, or symbolically:

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Example 3 Find the equation of the line tangent to the function $f(x) = x^3 - x^2 - 4x + 6$ when $x = 2$. Graph the function and the tangent line on your calculator.

The Velocity Problem

Suppose we need to find the velocity of a car at a particular instance in time. Usually, the velocity is not always constant, but changing continually. Instead of calculating the *instantaneous velocity* we instead can only calculate the *average velocity* over a particular time interval. The definition of **average velocity** is

$$\text{average velocity} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

Example 4 The distance, s, a car travels after t seconds is given in the following table:

t (sec)	0	0.2	0.4	0.6	0.8	1.	1.2	1.4	1.6	1.8	2
s (ft)	0	0.8	3.2	7.2	12.8	20	28.8	39.2	51.2	64.8	80

(a) Compare the average velocity over the time intervals [0.2, 0.4] and [1.8, 2].

(b) Estimate the *instantaneous* velocity at time $t = 1$ second.

Example 5 The distance the car travels in example 4 can be modeled by the function $s(t) = 20t^2$. Use $s(t)$ and a table of values to calculate the *instantaneous* velocity at $t = 1$, and compare the values with example 4.