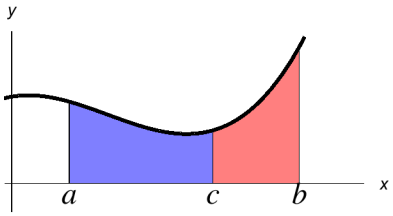
# 4.4 Properties of Definite Integrals

### Theorem 1

Let $f(x)$ be a differentiable function on the interval $[a, b]$. Then for any number $c$, $a \le c \le b$:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

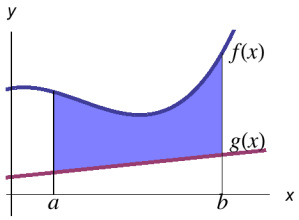



**Example 1** Set up the integral and calculate the area in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = -x + 6$.

## Area of a Region Bounded by Two Functions

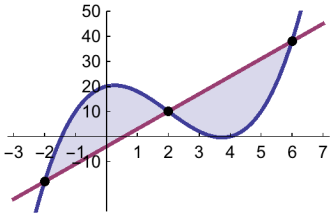
The area between two functions $f(x)$ and $g(x)$ where $f(x) \ge g(x)$ is given by

$$A = \int_a^b [f(x) - g(x)]\,dx$$




**Example 2** Find the area bounded by $f(x) = x^2$ and $g(x) = x + 2$

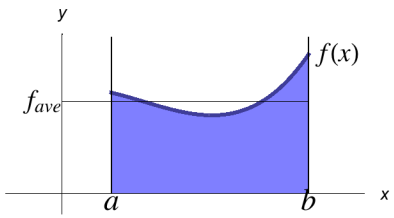
**Example 3** Set up the integral to find the area bounded by $f(x) = x^3 - 6x^2 + 3x + 20$ and $g(x) = 7x - 4$




## Average Value of a Function

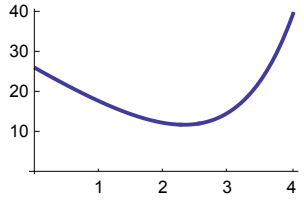
The *average value* of a function is given by

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$




**Example 4** Find the average value of the function $f(x) = x^2 - 4x + 7$ on the interval [1, 5]

**Example 5** The value of a stock varied over a 4 week period and can be approximated by the function $f(x) = e^x - 10x + 25$. Find the average value of the stock over the four week period.




**Example 6** Find $b$ such that the average value of the function $f(x) = x^2$ is 4 on the interval [1, $b$].