

4.3 The Fundamental Theorem of Calculus

Theorem: The Fundamental Theorem of Calculus (Part II)

Let $f(x)$ be a continuous function with any anti-derivative $F(x)$ over $[a, b]$, then

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_a^b f(x) dx = F(b) - F(a)$$

Example 1 Evaluate the definite integral $\int_2^5 (x^2 + 4x + 2) dx$

Example 2 Find the area under the curve $f(x) = 3e^{-0.2x}$ from $x = -2$ to $x = 4$. Round to three decimal places.

Example 3 Evaluate the integral $\int_1^4 (x^2 - 7) dx$.

Example 4 Find the area bounded by $f(x) = -3x^2 - 6x + 24$ and the x -axis on the interval $[0, 4]$.

Example 5 The monthly marginal profit of a company is modeled by $P'(t) = 300e^{0.05t} - 2.8t - t^2 - 300$ where t is in months, and P is in thousands of dollars. Calculate the total profit over the first 3 years.

Additional Sample Problems

1. Find b such that the area under the curve $f(x) = 0.6e^{-0.5x}$ on the interval $[0, b]$ is equal to 1.
2. The marginal cost of an item is given by $C'(x) = 0.015x + 12$. If the fixed costs are \$5000, how many items can be manufactured to keep total costs below \$20000?
3. The velocity of a particle is given by $v(t) = t^2 + 3t - 3$. How far has it moved when its acceleration is 7 ft/s^2 .