4.1 Anti-differentiation

Anti-differentiation

The function $f(x)$ has an **anti-derivative** $F(x) + C$ if $\frac{d}{dx}[F(x) + C] = f(x)$ for all x. The notation for the anti-derivative is

$$\int f(x)\,dx = F(x) + C$$

and the left-side of the expression is called an **indefinite integral**.

Example 1 Show that $F(x) = x^3 + 2x^2 - 4x + 5$ is an anti-derivative of $f(x) = 3x^2 + 4x - 4$.

Properties and Rules for Anti-Derivatives

Power Rule

$$\int x^n\,dx =$$

Constant Factor Property

$$\int c \cdot f(x)\,dx =$$

Example 2 Find the indefinite integrals

a. $\int 8x\,dx$

b. $\int x^5\,dx$

c. $\int \sqrt{x}\,dx$

d. $\int \frac{-6}{x^3}\,dx$

e. $\int \frac{5}{x\sqrt[4]{x}}\,dx$

f. $\int \frac{1}{x}\,dx$

Natural Logarithm Rule

$$\int \frac{1}{x}\,dx = \ln(x) + C \quad \text{if } x > 0, \quad \text{or}$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \quad \text{for all } x \text{ except } x \neq 0$$

Natural Exponential Rule

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

Example 3 Evaluate $\int e^{-2x}\,dx$

Sum and Difference Property

$$\int [f(x) \pm g(x)] = \int f(x)\,dx \pm \int g(x)\,dx$$

Example 4 Find the indefinite integrals:

a. $\int x^2(2x-3)^2\,dx$

b. $\int \frac{2xe^{5x}-4\sqrt{x}+5}{3x}\,dx$

c. $\int \frac{8}{2x+1}\,dx$

d. $\int 6(4x+1)^5\,dx$

Initial Conditions and the Constant of Integration

Example 6 Suppose the derivative of a function is $f'(x) = 2x - 5e^x$ and the function passes through the point (0, 2), that is $f(0) = 2$. Find the function.

Example 7 The velocity of a particle traveling on the number line at any time t is given by $v(t) = -4t + 8$. If the particle is at position 20 at time $t = 2$ find it's location at $t = 3$. Recall $s'(t) = v(t)$.