

4.1 Anti-differentiation

Anti-differentiation

The function $f(x)$ has an **anti-derivative** $F(x) + C$ if $\frac{d}{dx}[F(x) + C] = f(x)$ for all x . The notation for the anti-derivative is

$$\int f(x) dx = F(x) + C$$

and the left-side of the expression is called an **indefinite integral**.

Example 1 Show that $F(x) = x^3 + 2x^2 - 4x + 5$ is an anti-derivative of $f(x) = 3x^2 + 4x - 4$.

Properties and Rules for Anti-Derivatives

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Constant Factor Property

$$\int c \cdot f(x) dx = c \int f(x) dx$$

Example 2 Find the indefinite integrals

a. $\int 5 dx$

b. $\int x^5 dx$

c. $\int \sqrt{x} dx$

d. $\int \frac{-4}{x^3} dx$

e. $\int \frac{5}{x\sqrt[4]{x}} dx$

f. $\int \frac{1}{x} dx$

Natural Logarithm Rule

$$\int \frac{1}{x} dx = \ln(x) + C \text{ if } x > 0, \text{ or}$$

$$\int \frac{1}{x} dx = \ln |x| + C \text{ for all } x \text{ except } x \neq 0$$

Natural Exponential Rule

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

Example 3 Evaluate $\int e^{-2x} dx$

Sum and Difference Property

$$\int [f(x) \pm g(x)] = \int f(x) dx \pm \int g(x) dx$$

Example 4 Find the indefinite integrals:

a. $\int x^2(2x - 3)^2 dx$

b. $\int \frac{2xe^{5x-4}\sqrt{x}+5}{3x} dx$

c. $\int \frac{8}{2x+1} dx$

d. $\int 6(4x + 1)^5 dx$

Initial Conditions and the Constant of Integration

Example 6 Suppose the derivative of a function is $f'(x) = 2x - 5e^x$ and the function passes through the point $(0, 2)$, that is $f(0) = 2$. Find the function.

Example 7 The velocity of a particle traveling on the number line at any time t is given by $v(t) = -4t + 8$. If the particle is at position 20 at time $t = 2$ find its location at $t = 3$. Recall $s'(t) = v(t)$.