

## 3.6 Elasticity of Demand

### Definition: Relative Change

The *relative change* of a quantity  $q$  is given by  $\frac{\Delta q}{q}$ .

To analyze whether a small change in the price of a commodity affects the change in demand (quantity) and thus revenue, we can compare the relative change in quantity to the relative change in price. Given the demand function  $D = q(x)$  where  $x$  is the price of a commodity

$$\begin{aligned} \frac{\text{relative change in quantity}}{\text{relative change in price}} &= \frac{\frac{\Delta q}{q}}{\frac{\Delta x}{x}} \\ &= \frac{\Delta q}{q} \cdot \frac{x}{\Delta x} \\ &= \frac{\Delta q}{\Delta x} \cdot \frac{x}{q} \\ &\approx \frac{dq}{dx} \cdot \frac{x}{q} \\ &\approx \frac{x D'(x)}{D(x)} \quad \blacksquare \end{aligned}$$

### Elasticity of Demand

The last expression is referred to the *elasticity of demand*. However, since this value is generally always negative, elasticity of demand is defined as



$$E(x) = \frac{-x D'(x)}{D(x)}$$

This can also be written as  $E(x) = -x \frac{d}{dx}[\ln(D(x))]$ .

Note:

- If  $E < 1$ , **inelastic**, an increase in price ( $x$ ) results in a smaller relative decrease in demand: raise price to increase revenue.
- If  $E > 1$ , **elastic**, an increase in price ( $x$ ) results in a larger relative decrease in quantity demanded: lower price to increase revenue.
- If  $E = 1$ , an increase or decrease in price results in a corresponding decrease or increase in demand. A change will not result in an increase of revenue.

**Example 1** The demand function for a commodity is  $D(x) = x \sqrt{100 - x}$ . Find the elasticity at a price point of \$85. Should the company increase or decrease price to increase revenue? At what price point is there unitary elasticity?

**Example 2** The demand function for a newspaper is  $D(x) = \frac{100000}{1+3e^{0.7x}}$ . The current price of the newspaper is \$1.75. Find the current demand for the newspaper, the revenue, and the elasticity of demand at \$1.75. Should the publisher increase or decrease the paper's price to increase revenue? What price maximizes revenue and what is that revenue.