3.3 Applications of Exponential Growth

Uninhibited Growth

Suppose the rate of growth of a population is proportional to the size of the population. Then,

$$\frac{dP}{dt} = k\,P$$

where k is the constant of proportionality.

Show that the exponential function $P = P_0\, e^{k\,t}$ satisfies the rate equation. (Note: P_0 is the initial population, or the population when $t = 0$.)

Example 1 An investment of $20000 is growing at a continuous rate of 2.7% per year. Find the value of the investment in 4 years. How long will it take the investment to double?

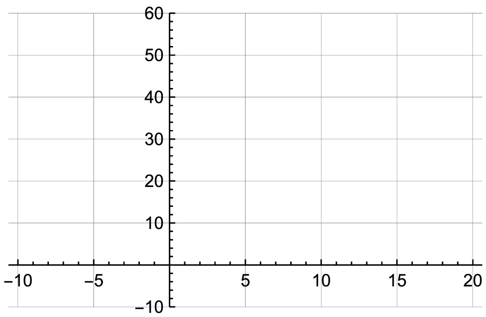
Example 2 World population is growing approximately 1.6% per year. What is the doubling time of the world population assuming the growth rate is constant?

Inhibited Growth (Or Limited Growth)

The **logistic equation** is a model for inhibited growth:

$$P(t) = \frac{L}{1+b\,e^{-k\,t}}$$

Example 3 Graph the logistic function $P(t) = \frac{50}{1+4\,e^{-0.5\,t}}$ using the window $X[-10, 20]$ and $Y[-10, 60]$.



Example 4 Find the rate of growth when $t = 0$, $t = 5$, and $t = 15$. Where is the rate of growth the greatest?

The Learning Curve

Example 5 A person memorizing all 272 words in the Gettysburg Address is given by $W(t) = 272\left(1 - e^{-0.25\,t}\right)$ where t is time in days and W is the number of words.

a) Graph the function using the window $[0, 20] \times [0, 300]$.

b) Find and interpret: $W(1)$ and $W(10)$

c) How long will it take to learn at least 270 words?

d) At what rate are they learning words on day 1; on day 10.