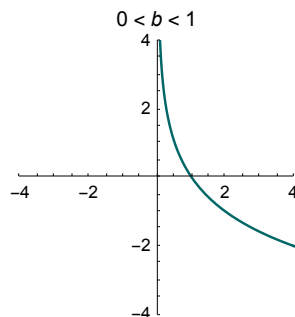
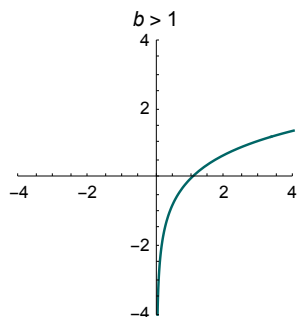


3.2 Derivatives of Logarithm Functions

Properties of Logarithms

- $x = b^y \implies \log_b(x) = y$
- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
- $\log_b(x^n) = n \log_b(x)$
- $\log_b(1) = 0$
- $\log_b(b) = 1$
- $\log_b(b^x) = x$
- $b^{\log_b(x)} = x$
- $\log_e(x) = \ln(x)$ and $\log_{10}(x) = \log(x)$
- $\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$ or $\log_b(a) = \frac{\ln(a)}{\ln(b)} = \frac{\log(a)}{\log(b)}$

Graphs of Logarithm Functions: $f(x) = \log_b(x)$



Derivative of the Natural Logarithm Function

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$$

Example 1 Find the derivative of $f(x) = x^4 \ln(x)$

Example 2 Find the derivative of $g(x) = \sqrt{\ln(x) + x}$

The Chain Rule with the Natural Logarithm Function

$$\frac{d}{dx}[\ln(f(x))] = \frac{1}{f(x)} \cdot f'(x)$$

or,

$$\frac{d}{dx}[\ln(u)] = \frac{1}{u} \cdot \frac{du}{dx}$$

Example 3 Find the derivative of $h(x) = 3 \ln(x^3 - 4x + 5)$

Example 4 Find the derivative of $f(x) = \ln(\ln(2x + 1))$

Example 5 Find the derivative of $G(x) = \ln\left(\frac{2x^2 + 7x - 2}{x^2 - 4x}\right)$

Example 6 Find the derivative of $C(x) = \ln(\sqrt{3x^2 + 5})$

Challenge Problem Find $\frac{dy}{dx}$ for the implicit function $2xy^2 + \ln(xy) + e^{x-y} = 3$ at the point $(1, 1)$.