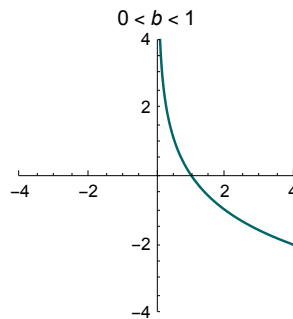
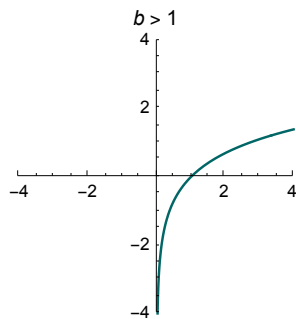


3.2 Logarithm Functions

Properties of Logarithms

1. $x = b^y \implies$
2. $\log_b(xy) =$
3. $\log_b\left(\frac{x}{y}\right) =$
4. $\log_b(x^n) =$
5. $\log_b(1) =$
6. $\log_b(b) =$
7. $\log_b(b^x) =$
8. $b^{\log_b(x)} =$
9. $\log_e(x) =$ and $\log_{10}(x) =$
10. $\log_b(a) =$

Graphs of Logarithm Functions: $f(x) = \log_b(x)$



Derivative of the Natural Logarithm Function

$$\frac{d}{dx}[\ln(x)] =$$

Example 1 Find the critical numbers for $f(x) = x^4 \ln(x)$.

The Chain Rule with the Natural Logarithm Function

$$\frac{d}{dx}[\ln(f(x))] = \frac{1}{f(x)} \cdot f'(x)$$

or,

$$\frac{d}{dx}[\ln(u)] = \frac{1}{u} \cdot \frac{du}{dx}$$

Example 2 Find the derivative of $h(x) = \ln(x^3 - 4x + 5)$.

Example 3 Find the derivative of $f(x) = \ln(x^5)$.

Example 4 Find the derivative of $G(x) = \ln\left(\frac{\sqrt{x^2+1}}{(x+3)^2(x-4)}\right)$

Example 5 Write the *elasticity of demand*, $E(x) = \frac{-x D'(x)}{D(x)}$ formula in terms of the natural log. Find the elasticity for the demand function $D(x) = x\sqrt{100-x}$ at the price point $x = 85$.

Challenge Problem Find $\frac{dy}{dx}$ for the implicit function $2xy^2 + \ln(xy) + e^{x-y} = 3$ at the point $(1, 1)$.