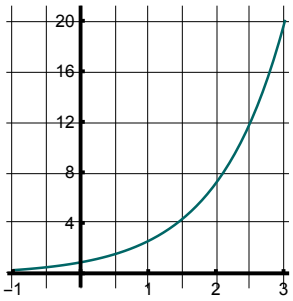
3.1 Exponential and Logarithmic Functions

Review of Exponents

Rules of Exponents

1. $b^x \cdot b^y = b^{x+y}$
2. $\frac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$

Example 1 For the function $f(x) = e^x$, estimate the slope at $x = 2$, that is $f'(2)$.



Derivative of the Natural Exponential Function

Let $f(x) = e^x$ be the natural exponential function where $e \approx 2.71828\ldots$. The derivative of f is given by

$$\frac{d}{dx}[e^x] = e^x$$

Example 2 Find the slope of the tangent line to $f(x) = 2e^x - 5x^2$ when $x = 2$.

The Chain Rule for the Natural Exponential Function

$$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x)$$

or,

$$\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$$

Example 3 Find the derivative of: $f(x) = 3e^{x^2+3x-1}$

Example 4 Find the derivative of $f(x) = x^3 e^{4x}$.

Example 5 Find the critical numbers for $g(x) = \frac{e^x}{e^{-x}-1}$.

Example 6 Show that the inflection points for the "bell curve" $f(x) = e^{-x^2}$ are at $x = \pm \frac{1}{\sqrt{2}} \approx 0.707$.