2.8 Implicit Differentiation

An explicit function is a function that can be written as a function in a single variable, e.g., $f(x) = x^2 + 5x - 5$. An implicit function is a function such as $2x^2 y - 5y + 2 = x^2$. How can you find $\frac{dy}{dx}$?

Implicit Differentiation

Assume y is a function of x, then $\frac{d}{dx} y = y' = \frac{dy}{dx}$.

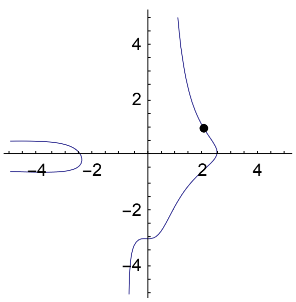
Power Rule for Implicit Differentiation

If y is a function of x then,

$$\frac{d}{dx} y^n = n y^{n-1} \cdot \frac{dy}{dx}, \quad \text{or}$$
$$= n y^{n-1} \cdot y'$$

Example 1 Find $\frac{dy}{dx}$ for the implicit function $y^3 = x^2 + 4y$, to find y'.

Example 2 Find $\frac{dy}{dx}$ for the implicit function $2x^2 + x^3 y^2 - 4y = 12$, and find the value of $\frac{dy}{dx}$ at (2, 1)



Example 3 If p is the price of an item, then the demand for that item is $x = p\sqrt{41 - p^2}$ where x is the number of units. Find the demand x, *and* $\frac{dp}{dx}$, when the price if \$5.

Related Rates

Example 4 Assume two functions are both a function of t, i.e. $x(t)$ and $y(t)$ such that $4x + x^2y^2 = y + 10$. Find $\frac{dy}{dt}$ when $x = 1$, $y = 3$, and $\frac{dx}{dt} = -2$

Example 5 Suppose the price, p in dollars, of coffee mugs and the number of sales, x, are related by the equation $xp^2 + 3px = 4000$. Assume p and x are functions of time measured in days. Find $\frac{dx}{dt}$ when $p = 5$ and $\frac{dp}{dt} = 0.25$.