2.7 Implicit Differentiation

An explicit function is a function that can be written as a function in a single variable, e.g., $f(x) = x^2 + 5x - 5$. An implicit function is a function such as $2x^2 y - 5y + 2 = x^2$. How can you find $\frac{dy}{dx}$?

Implicit Differentiation

Assume y is a function of x, then $\frac{d}{dx} y = y' = \frac{dy}{dx}$.

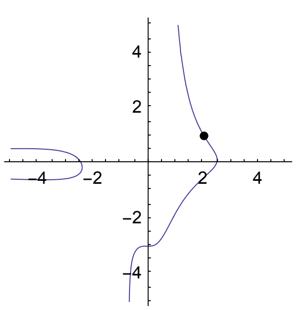
Power Rule for Implicit Differentiation

If y is a function of x then,

$$\frac{d}{dx} y^n = n\, y^{n-1} \cdot \frac{dy}{dx}, \quad \text{or}$$
$$= n\, y^{n-1} \cdot y'$$

Example 1 Find $\frac{dy}{dx}$ for the implicit function $y^3 = x^2 + 4y$, to find y'.

Example 2 Find $\frac{dy}{dx}$ for the implicit function $2x^2 + x^3 y^2 - 4y = 12$, and find the value of $\frac{dy}{dx}$ at (2, 1)



Example 3 If p is the price of an item, then the demand for that item is $x = p\sqrt{41 - p^2}$ where x is the number of units. Find the demand x, and $\frac{dp}{dx}$, when the price if \$5.

Related Rates

Example 4 Assume two functions are both a function of t , i.e. $x(t)$ and $y(t)$ such that $4x + x^2y^2 = y + 10$. Find $\frac{dy}{dt}$ when $x = 1$, $y = 3$, and $\frac{dx}{dt} = -2$

Example 5 The base of a 10-foot ladder leaning against a wall is being pulled from the wall at a rate of 1.0 feet per second. Find the rate of the top of the ladder when

- **a.** The top is 8 feet above the ground.
- **b.** When the base is 9 feet from the wall.
- **c.** When the ladder hits the ground.