

2.4 Absolute Extrema (or Global Extrema)

Absolut Extreme Values

The absolute extreme value of a function on an interval I is the value that is the largest (or smallest) value of the entire function on that interval. That is:

$f(c)$ is an **absolute maximum** if $f(c) \geq f(x)$ for all x in I .

$f(c)$ is an **absolute minimum** if $f(c) \leq f(x)$ for all x in I .

Example 1 Find the absolute extrema of the function $f(x) = \frac{1}{2}x^2 - 3x + 4$ on the interval $[0, 4]$

Steps to Find Extreme Values

Suppose f is continuous on a closed interval $[a, b]$

1. Find all the critical numbers in $[a, b]$ (when $f'(x) = 0$ or $f'(x)$ is undefined.)
2. List the critical numbers and the **endpoints**: $a, c_1, c_2, c_3, \dots, b$
3. Evaluate $f(x)$ at each value in step 2: $f(a), f(c_1), f(c_2), \dots, f(b)$
4. The largest value in step 3 is the absolute maximum and the smallest value is the absolute minimum.

Example 2 Find the absolute extrema of the function $f(x) = -x^3 - 4x^2 + 3x + 18$ on the Interval $[-2, 4]$

Example 3 Find the absolute extrema for the function $f(x) = (x - 3)^5 (x + 6)^3$ on $[-4, 4]$.

Example 4 Find the absolute extrema of $f(x) = -4x^2 + 2x - 7$ on the interval $[0, 2]$

What can be said if there is only one critical number c in the interval and $f''(c) \neq 0$?