

## 2.3 Sophisticated Graphing

### Concepts from Pre-Calculus

1. Domain and Range of a Function
  - 1.1. Domain: all possible  $x$ -values
  - 1.2. Range: all possible  $y$ -values (often difficult; usually need some later techniques)
2. Intercepts
  - 2.1.  $x$ -intercepts: whenever  $f(x) = 0$
  - 2.2.  $y$ -intercepts:  $y = f(0)$
3. Tool-kit functions

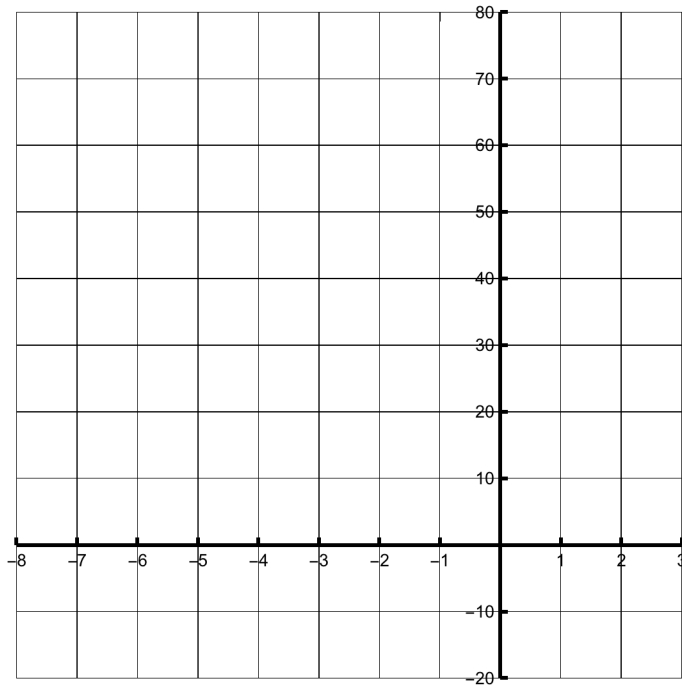
Knowing the general shape of basic functions: Linear, quadratic, power functions  $x^n$ , root functions  $\sqrt[n]{x}$ , reciprocal  $\frac{1}{x}$ , etc.

### Concepts from Calculus (and some from Pre - Calculus)

1. Asymptotes and Holes
  - 1.1. A reduced rational function  $f(x) = \frac{P(x)}{Q(x)}$  has vertical asymptotes  $x = c$  whenever  $Q(c) = 0$ . Use  $\lim_{x \rightarrow c^-} f(x)$  and  $\lim_{x \rightarrow c^+} f(x)$  to find the behavior of the vertical asymptotes.
  - 1.2. A reduced rational function  $f(x) = \frac{P(x)}{Q(x)}$  has horizontal asymptotes  $y = L$  if  $\lim_{x \rightarrow -\infty} f(x) = L$  or  $\lim_{x \rightarrow +\infty} f(x) = L$  exist. Example:
 
$$f(x) = \frac{3x^2 + 2x - 5}{2x^2 + x - 1}$$
  - 1.3. A "hole" can occur if  $P(x)$  and  $Q(x)$  have a common factor, e.g. when  $f(x) = \frac{P(x)}{Q(x)}$  is not reduced. Example:  $f(x) = \frac{x+2}{x^2-4}$ .
  - 1.4. Slant asymptotes occur when the degree of  $P(x)$  is greater than that of  $Q(x)$ . Example  $f(x) = \frac{x^2+5x-3}{x-1}$
2. Critical Numbers and Local Extrema (possible extrema)
  - 2.1.  $f'(x) = 0$ , or
  - 2.2.  $f'(x)$  is undefined

Use either the first or second derivative test to determine the local extrema.
3. Intervals of Increasing or Decreasing
  - 3.1. Intervals on which  $f'(x) > 0$  are increasing
  - 3.2. Intervals on which  $f'(x) < 0$  are decreasing
4. Inflection Points (possible)
  - 4.1.  $f''(x) = 0$ , or
  - 4.2.  $f''(x)$  is undefined
5. Intervals of Concavity
  - 5.1. Intervals on which  $f''(x) > 0$  are concave up
  - 5.2. Intervals on which  $f''(x) < 0$  are concave down

**Example 1** Make an accurate sketch of  $f(x) = x^3 + 6x^2 + 9x + 54$  by indicating  $x$ -intercepts,  $y$ -intercepts, asymptotes, critical values, inflection points, etc.



**Example 2** Make an accurate sketch of the rational function  $f(x) = \frac{x^2+3}{x+1}$  indicating intercepts, asymptotes, local extrema, etc.

