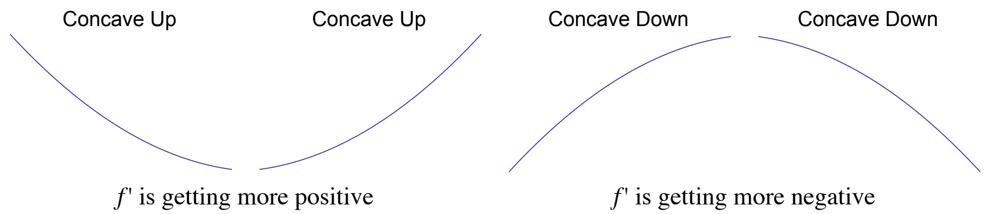
2.2 The Second Derivative Test

Concavity measures whether a function's slope is increasing (getting steeper) or decreasing (getting less steep).



Theorem: Test for Concavity

1. If $f''(x) > 0$ for all x in an open interval I, the graph of f is **concave up**.
2. If $f''(x) < 0$ for all x in an open interval I, the graph of f is **concave down**.

Example 1 Determine if the function $f(x) = -x^3 + 6x + 3$ is concave up or concave down at $x = 2$.

Theorem 2: The Second Derivative Test

Suppose that f is differentiable for all x in an open interval (a, b) and that there is a critical number c in (a, b) for which $f'(c) = 0$.
1. $f(c)$ is a relative minimum if $f''(c) > 0$.
2. $f(c)$ is a relative maximum if $f''(c) < 0$.
If $f''(c) = 0$ then use the first derivative test to determine whether $f(c)$ is a relative extrema.

Example 2 Find the relative extrema for the function $f(x) = -x^3 - 3x^2 + 9x + 12$

Example 3 Determine where the function $f(x) = x^3 - 3x^2 - 4x + 7$ changes from concave up to concave down.

Theorem 3: Inflection Points

If a function f has a point of inflection at c then $f''(c) = 0$ or $f''(c)$ does not exist.

Note: This does nat mean that if $f''(c) = 0$ or $f''(c) =$ undefined then an inflection point exists at c.

Example 4 Graph the function $f(x) = x^3 + 3x^2 - 24x + 15$ indicating relative extrema, inflection points, concavity, intervals of increasing and decreasing, and intercepts when possible.