2.1 Relative Extrema and The First Derivative Test

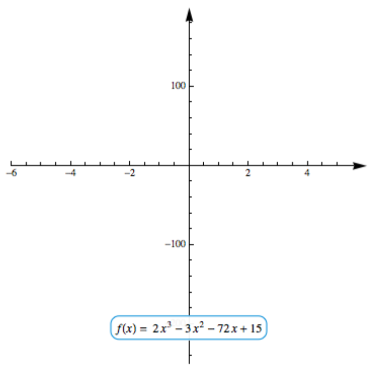
Theorem 1: Increasing and Decreasing Intervals

A function is **increasing** on an open interval I if $f'(x) > 0$ for all x in I.
A function is **decreasing** on an open interval I if $f'(x) < 0$ for all x in I.

Definition: Critical Number

A **critical number** of a function f is any number c in the domain of f for which $f'(c) = 0$ or $f'(c)$ is undefined.

Example 1 Find the critical numbers for $f(x) = 2x^3 - 3x^2 - 72x + 15$, and find the intervals on which f is increasing and decreasing.



Local Extrema

A critical number c gives a **local maximum** if $f(c) \geq f(x)$ for all x "near" c.
A critical number c gives a **local minimum** if $f(c) \leq f(x)$ for all x "near" c.

Example 2 Find the critical numbers for $f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - 2x^2 - 8x + 3$ and local extrema. Does each critical number give a local extrema?

Example 3 Find the local extrema for $f(x) = \frac{x^2-3x+4}{2x^2+1}$, round to three decimal places.

Example 4 Find all critical numbers and local extrema for $f(x) = x^2 - 3\sqrt[3]{x^2}$.