# 2.1 Relative Extrema and The First Derivative Test

A function is **increasing** on an interval if $f(x_2) > f(x_1)$ whenever $x_2 > x_1$ on the interval, and **decreasing** when $f(x_2) < f(x_1)$ whenever $x_2 > x_1$ on the interval.

**Example 1** Is $f(x) = \sqrt{x}$ increasing or decreasing on the interval [1, 4]? Find the slope of the secant line on that interval.

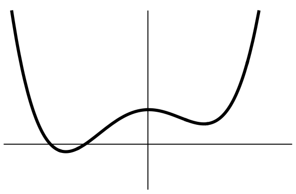
### Theorem 1: Increasing and Decreasing Intervals

A function is **increasing** on an open interval $I$ if $f'(x) > 0$ for all $x$ in $I$.
A function is **decreasing** on an open interval $I$ if $f'(x) < 0$ for all $x$ in $I$.

### Definition: Critical Number

A **critical number** of a function $f$ is any number $c$ in the domain of $f$ for which $f'(c) = 0$ or $f'(c)$ is undefined.

**Example 2** Find the critical numbers for $f(x) = \frac{3}{4}x^4 + x^3 - 9x^2 + 40$, and find the intervals on which $f$ is increasing and decreasing.



### Definition: Local Extrema

A critical number $c$ gives a **local maximum** if $f(c) \geq f(x)$ for all $x$ "near" $c$.
A critical number $c$ gives a **local minimum** if $f(c) \leq f(x)$ for all $x$ "near" $c$.

### Theorem 2

If a function $f$ has a relative extrema at $x = c$, then either $f'(c) = 0$ or $f'(c)$ does not exist.

**Example 3** Find the critical numbers for $f(x) = (x^2 - 3x)^3$ and the critical points. Does each critical number give a local extrema?

## The First Derivative Test

Suppose $x = c$ is a critical number of $f$. Then,

1. If $f'(x)$ changes sign from positive to negative at $c$, then $f(c)$ is a local maximum.
2. If $f'(x)$ changes sign from negative to positive at $c$, then $f(c)$ is a local minimum.
3. If $f'(x)$ does not change sign at $c$, then $f(c)$ is nether a local max or local min.

**Example 4** Find the local extrema for $f(x) = \frac{x^2-3x-3}{x^2+1}$, and determine if the critical numbers give a local max, local min, or neither. Graph to verify your results.

**Example 5** Find all critical numbers and local extrema for $f(x) = x^2 - 3\sqrt[3]{x^2}$.

**Example 6** Find the critical numbers for the quadratic $f(x) = ax^2 + bx + c$