

## 1.5 Differentiation: Power, Sum and Difference Rules

**Example 1** Find  $\frac{d}{dx}[x^6]$

$$\begin{aligned}\frac{d}{dx} x^6 &= \lim_{h \rightarrow 0} \frac{(x+h)^6 - x^6}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^6 + 6hx^5 + 15x^4h^2 + 20x^3h^3 + 15x^2h^4 + 6xh^5 + h^6 - x^6}{h} \\ &= \lim_{h \rightarrow 0} (6x^5 + 15x^4h + 20x^3h^2 + 15x^2h^3 + 6xh^4 + h^5) \\ &= 6x^5\end{aligned}$$

### The Power Rule

$$\frac{d}{dx}[x^k] = kx^{k-1}$$

**Example 2** Find  $f'(x)$  for  $f(x) = \sqrt{x}$ .

**Example 3** Find  $\frac{d}{dx}\left[\frac{1}{x^3}\right]_{x=2}$ .

### The Constant Factor Rule

$$\begin{aligned}\frac{d}{dx}[cf(x)] &= c \frac{d}{dx}[f(x)] \\ &= cf'(x)\end{aligned}$$

**Example 4** Find  $\frac{d}{dx}[3x^5]$

**Example 5** Find  $f'(x)$  for  $f(x) = -4\sqrt[3]{x^2}$ .

**Example 6** Find  $\frac{d}{dx}[6]$

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### The Sum and Difference Rule

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

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**Example 7** Find  $\frac{d}{dx}[4x^3 - 2x^2 + 7x + 3]$

**Example 8** Find  $f'(x)$  for the function  $f(x) = \frac{2x^2 + \sqrt{x} - 3}{5x}$ , and  $f'(1)$ .

**Example 9** Find where the line tangent to the graph of the function  $f(x) = x^3 - 4x^2 + 4x + 1$  is horizontal.