1.4 The Derivative

Definition: The derivative of a function $y = f(x)$ is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists. If $f'(x)$ exists, then we say f is differentiable at x.

Notation for the derivative: $y' = f'(x) = D_x[f] = \frac{dy}{dx} = f_x(x)$

Example 1 Find the derivative of $f(x) = x^2$ and find $f'(-2)$, $f'(0)$, and $f'(3)$

Example 2 Find the derivative of $f(x) = \sqrt{x} + 3$

Example 3 Find the derivative of $y = -5x + 6$

Example 4 Find the equation of the line tangent to the curve of $y = \frac{6}{x}$ when $x = 2$.

Note: A differentiable function must be smooth and continuous.

Example 5 Determine if the following function is differentiable at $x = 2$: $f(x) = \begin{cases} x^2 & x \le 2 \\ -2x + 8 & x > 2 \end{cases}$.

Example 6 Find values a and b so that the function $f(x) = \begin{cases} \sqrt{x} & x \le 4 \\ ax + b & x > 4 \end{cases}$ is differentiable everywhere.