

1.3 Average Rates of Change

Example 1 The location of a toy car from its starting point after t seconds is given by $s(t) = -0.5t^2 + 6t$ (in feet). Find its average speed from $t = 2$ seconds to $t = 4$ seconds.

Definition: Average Rate of Change

The *average rate of change* of the function f from $x = x_1$ to $x = x_2$ is

$$\text{ARC} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Example 2 Find the average rate of change of the function $f(x) = x^2$ on the interval (a) $[3, 5]$, (b) $[3, 4]$, (c) $[3, 3.1]$ (d) $[3, 3.01]$, (e) $[3, 3.001]$ etc. What value is the average rate of change approaching?

The Difference Quotient

The average rate of change of a function f from x to $x + h$ is given by the *difference quotient*

$$\text{ARC} = \frac{f(x+h) - f(x)}{h}$$

Example 3 Find a simplified expression of the difference quotient using the function $f(x) = 3x^2 + 2x - 5$

Example 4 Find a simplified expression of the difference quotient using the function $f(x) = \frac{3}{x}$

Example 5 Simplify the difference quotient using the function $f(x) = \sqrt{x}$, and make a table of values for $x = 4$ and $h = \{2, 1, 0.1, 0.01, 0.001, 0.0001\}$.