1.2 Algebraic Limits and Continuity

Example 1 a) If $f(x) = 5$, find $\lim_{x \to 2} f(x)$ b) if $g(x) = x$, find $\lim_{x \to 3} g(x)$

Limit Properties

1. $\lim_{x \to a} c = c$
2. $\lim_{x \to a} x = a$
3. If $\lim_{x \to a} f(x) = L$, then $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n = L^n$
4. If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, then $\lim_{x \to a} [f(x) \pm g(x)] = L \pm M$
5. $\lim_{x \to a} [f(x) \cdot g(x)] = L \cdot M$
6. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ (as long as $M \neq 0$)
7. $\lim_{x \to a} c \cdot f(x) = c \lim_{x \to a} f(x) = c \cdot L$

Example 2 Find $\lim_{x \to 2} \left(x^2 - 3x + 5\right)$

Example 3 Find $\lim_{x \to 2} \frac{2x^2 - 3x - 2}{x - 2}$

Example 4 Find $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$

Continuity

Laymen's Definition: A function is *continuous* if the entire graph of the function can be made without lifting your pencil.

Definition: Continuity

A function f is **continuous** at $x = a$ if

1. $f(a)$ exists,
2. $\lim_{x \to a} f(x)$ exists, and
3. $\lim_{x \to a} f(x) = f(a)$

Example 5 Is the function $f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{for } x \neq 3 \\ 2 & \text{for } x = 3 \end{cases}$ continuous at $x = 3$. If not, find the value of $f(3)$ necessary for f to be continuous at $x = 3$.

Example 6 Is the function $f(x) = \begin{cases} 3x - 2 & \text{for } x \leq 2 \\ \frac{x^2-4}{x-2} & \text{for } x > 2 \end{cases}$ continuous at $x = 2$.

Example 7 Find the value of a which will make the function f continuous at $x = -1$; $f(x) = \begin{cases} \frac{x^3+1}{x+1} & \text{for } x < -1 \\ a x^2 + 2 & \text{for } x \geq -1 \end{cases}$.