

Math 148 Business Calculus Notes

1.1 Limits: Numerical and Graphical

Example 1 What happens to the function $f(x) = \frac{e^x - 1}{x}$ when x gets close to 0?

Definition: Limit

As x approaches a , the **limit** of $f(x)$ is L , and is written

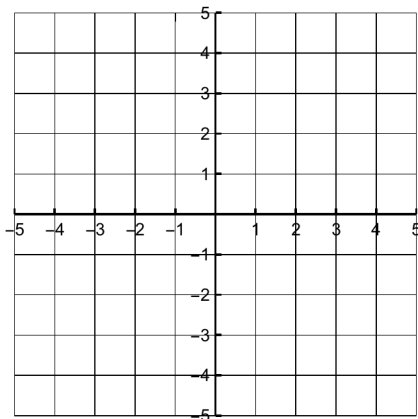
$$\lim_{x \rightarrow a} f(x) = L$$

if the value of $f(x)$ is close to L for all x 's arbitrarily close to a . Note: L must be a finite number.

Example 2 Use a table of values to estimate $\lim_{x \rightarrow -2} \frac{2x^3 + 16}{x + 2}$ and verify by graphing the function.

One Sided Limits

Example 3 Graph the function: $f(x) = \begin{cases} 2x - 1 & \text{if } x < 2 \\ -x + 3 & \text{if } x \geq 2 \end{cases}$.



Find:

(a) $f(2)$

(b) $\lim_{x \rightarrow 2^-} f(x)$

(c) $\lim_{x \rightarrow 2^+} f(x)$

(d) $\lim_{x \rightarrow 2} f(x)$

Theorem

If $\lim_{x \rightarrow a^-} f(x) = L = \lim_{x \rightarrow a^+} f(x)$, then $\lim_{x \rightarrow a} f(x) = L$.

Example 4 Determine if $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$ exists. If it does exist, find the limit.

Limits Involving Infinity

Example 5 Given the function $f(x) = \frac{2}{x-3}$ find

- (a) $\lim_{x \rightarrow 3^-} f(x)$
- (b) $\lim_{x \rightarrow 3^+} f(x)$

Example 6 Given the function $g(x) = \frac{1}{x^2}$ determine if $\lim_{x \rightarrow 0} g(x)$ exists. If it does exist, find the limit.

Example 7 Given the function $h(x) = \frac{2x-3}{3x+6}$ find

- (a) $\lim_{x \rightarrow -\infty} h(x)$
- (b) $\lim_{x \rightarrow +\infty} h(x)$