4.5 Distribution of Wealth - The Gini Coefficient

The distribution of wealth can be represented by a percent function $y = f(x)$, where x is the percentage of the population and y is the percentage of wealth that population has. That is, if $y = f(0.5) = 0.23$, this means that 50% of the population only has 23% of the wealth. The domain for x is [0, 1] and the range of y is also [0, 1]. In an **ideal** population, 10% of the people would have 10% of the wealth, 40% would have 40% of the wealth, etc. This is represented by the function $f(x) = x$ and is called a *Lorenz function*. Notice $f(0) = 0$ and $f(1) = 1$. Figure 1 shows the graph of this perfectly distributed wealth. In reality, wealth is not distributed evenly, i.e., the top 5% may have 50% of the wealth (the bottom 95% has the other 50%). Figures 2 and 3 shows the Lorenz function for less equitable distributions.

Fig. 1

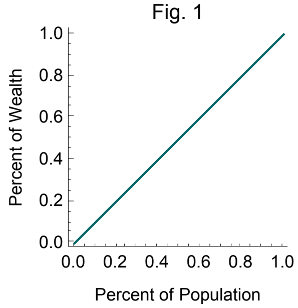


Fig. 2

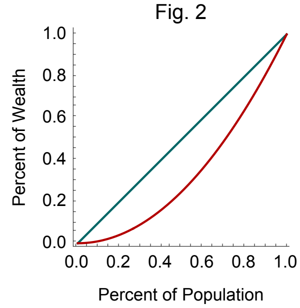
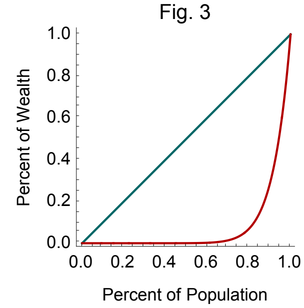


Fig. 3



Notice that the area between the perfect equality line and the Lorenz function is small when the distribution is close to equitable, and large when the distribution is inequitable. This area can have values from 0 for equality and 0.5 for absolute inequality (one person has ALL the wealth). The Gini coefficient is a measure of this inequality and is two times the area to make the values be between 0 and 1:

$$\text{Gini Coefficient} = 2\int_0^1 (x - f(x))\, dx$$

Gini Coefficient for World Geographic Areas

Example 1 Suppose a country has the Lorenz function $f(x) = x^{2.5}$.

(a) What percentage of wealth is owned by the bottom 70% of the population?

(b) What is the Gini coefficient for this country?
(The coefficient is usually multiplied by 100 to have values between 0 and 100).

Example 2 A country reported that the top 10% of the population has 90% of the wealth. Find the Lorenz function in the form $f(x) = x^n$ for this country and the corresponding Gini coefficient.

Example 3 If the Gini coefficient of a country is 45, find the Lorenz function and calculate how much wealth 50% of the population owns.