# 3.3-3.4 Applications of Exponential Growth and Decay

## Uninhibited Growth

Suppose the rate of growth of a population is proportional to the size of the population. Then,

$$\frac{dP}{dt} = k\,P$$

where $k$ is the constant of proportionality.

Show that the exponential function $P = P_0\,e^{k\,t}$ satisfies the rate equation. (Note: $P_0$ is the initial population, or the population when $t = 0$.)

**Example 1** An investment of $20000 is growing at a continuous rate of 2.7% per year. Find the value of the investment in 4 years. How long will it take the investment to double?

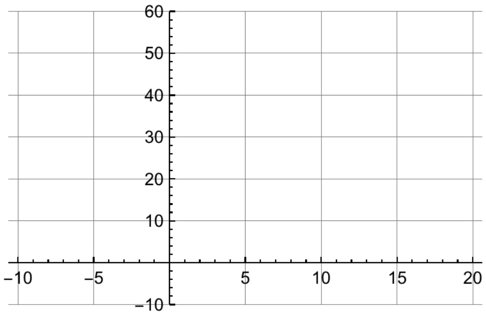
**Example 2** World population is growing approximately 1.6% per year. What is the doubling time of the world population assuming the growth rate is constant?

## Inhibited Growth (Or Limited Growth)

The **logistic equation** is a model for inhibited growth:

$$P(t) = \frac{L}{1+b\,e^{-kt}}$$

**Example 3** Graph the logistic function $P(t) = \frac{50}{1+4\,e^{-0.5t}}$ using the window $X[-10, 20]$ and $Y[-10, 60]$.




**Example 4** Find the rate of growth when $t = 0$, $t = 5$, and $t = 15$. Where is the rate of growth the greatest?

## Newton's Law of Cooling

Newton discovered the rate an object cools is proportional to the difference in the temperature of the object and the surrounding temperature, or $\frac{dT}{dt} = -k(T - C)$. Show that the solution to this differential equation is

$$T = a\,e^{-kt} + C$$

**Example 5** A piece of pottery is taken out of a kiln at a temperature of 1200° C, and placed into a cooling room with a constant temperature of 15° C. After 2 hours the temperature was 800° C. Find the temperature after 8 hours. How long will it take for the pottery to be 30° C?