

2.7 Elasticity of Demand

Definition: Relative Change

The *relative change* of a quantity q is given by $\frac{\Delta q}{q}$.

To analyze whether a small change in the price of a commodity affects the change in demand (quantity) and thus revenue, we can compare the relative change in quantity to the relative change in price. Given the demand function $D = q(x)$ where x is the price of a commodity

$$\begin{aligned} \frac{\text{relative change in quantity}}{\text{relative change in price}} &= \frac{\frac{\Delta q}{q}}{\frac{\Delta x}{x}} \\ &= \frac{\Delta q}{q} \cdot \frac{x}{\Delta x} \\ &= \frac{\Delta q}{\Delta x} \cdot \frac{x}{q} \\ &\approx \frac{dq}{dx} \cdot \frac{x}{q} \\ &\approx \frac{x D'(x)}{D(x)} \quad \blacksquare \end{aligned}$$

Elasticity of Demand

The last expression is referred to the *elasticity of demand*. However, since this value is generally always negative, elasticity of demand is defined as



$$E(x) = \frac{-x D'(x)}{D(x)}$$

Note:

- If $E < 1$, **inelastic**, an increase in price (x) results in a smaller relative decrease in demand: raise price to increase revenue.
- If $E > 1$, **elastic**, an increase in price (x) results in a larger relative decrease in quantity demanded: lower price to increase revenue.
- If $E = 1$, an increase or decrease in price results in a corresponding decrease or increase in demand. A change will not result in an increase of revenue.

Example 1 The demand function for a commodity is $D(x) = x \sqrt{100 - x}$. Find the elasticity at a price point of \$85. Should the company increase or decrease price to increase revenue? At what price point is there unitary elasticity?

Example 2 The demand function for tomato plants is $D(x) = \frac{2x+300}{10x+11}$. Find the elasticity of demand if the current price is \$3 per plant. Determine if an increase in price will increase or decrease revenue.